# Klasik Mantık
# ve
# Bulanık Mantığa Giriş

Rasim TEMUR

İstanbul Üniversitesi İnşaat Mühendisliği Bölümü

# Mantıksal Bağlar

## işaretler

| *İşaret* | *Anlamı* |
| --- | --- |
| $\neg$ | değil |
| $\wedge$ | ve |
| $\vee$ | veya |
| $\Rightarrow$ | ise |
| $\Leftrightarrow$ | sadece ve sadece |

# Mantıksal Bağlar

## tersini alma

| p | ¬p |
|---|---|
| D | Y |
| Y | D |

| p | ¬p |
|---|---|
| 1 | 0 |
| 0 | 1 |

$$|\neg p| = 1 - |p|$$

$|p|$ : p önermesinin gerçek değeri

# Mantıksal Bağlar

kesişim

| p | q | $p \wedge q$ |
|---|---|---|
| D | D | D |
| D | Y | Y |
| Y | D | Y |
| Y | Y | Y |

| p | q | $p \wedge q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

$$|p \wedge q| = \min\left[\,|p|\,,|q|\,\right]$$

$$|p \wedge q| = |p| \cdot |q|$$

$$|p \wedge q| = \max\left[\,0\,,|p| + |q| - 1\,\right]$$

# Mantıksal Bağlar

birleşim

| p | q | $p \vee q$ |
|---|---|---|
| D | D | D |
| D | Y | D |
| Y | D | D |
| Y | Y | Y |

| p | q | $p \vee q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

$$|p \vee q| = \max\left[\,|p|, |q|\,\right]$$

$$|p \vee q| = \min\left[\,1, |p| + |q|\,\right]$$

# Mantıksal Bağlar

## koşullu önermeler

| p | q | $p \Rightarrow q$ |
|---|---|---|
| D | D | D |
| D | Y | Y |
| Y | D | D |
| Y | Y | D |

| p | q | $p \Rightarrow q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 0 | 1 |

$$|p \Rightarrow q| = \min\left[1, 1 + |q| - |p|\right]$$

$$|p \Rightarrow q| = 1 - |p|(1 - |q|)$$

$$|p \Rightarrow q| \neq |q \Rightarrow p|$$

# Mantıksal Bağlar

## çift yönlü koşullu önermeler

| $p$ | $q$ | $p \Leftrightarrow q$ |
|---|---|---|
| D | D | D |
| D | Y | Y |
| Y | D | Y |
| Y | Y | D |

| $p$ | $q$ | $p \Leftrightarrow q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

$$|p \Leftrightarrow q| = |p| \cdot |q| + |\neg p| \cdot |\neg q|$$

# Mantıksal Bağlar

## çelişki

**Çelişki:** Basit önermelerinin doğruluk değeri ne olursa olsun yanlış olan bileşke önermedir

| p | $p \wedge \neg p$ |
|---|---|
| D | Y |
| Y | Y |

| p | $p \wedge \neg p$ |
|---|---|
| 1 | 0 |
| 0 | 0 |

# Mantıksal Bağlar

## totoloji

**Totoloji:** Basit önermelerinin doğruluk değeri ne olursa olsun doğru olan bileşke önermedir

| $p$ | $p \vee \neg p$ |
|---|---|
| D | D |
| Y | D |

| $p$ | $p \vee \neg p$ |
|---|---|
| 1 | 1 |
| 0 | 1 |

# Klasik Mantık

## yer değiştirme kuralları

| | | |
|---|---|---|
| Çift değilleme | $p \Leftrightarrow \neg\neg p$ | |
| Değişme | $(p \wedge q) \Leftrightarrow (q \wedge p)$ | $(p \vee q) \Leftrightarrow (q \vee p)$ |
| Birleşme | $[\, p \wedge (q \wedge r)\,] \Leftrightarrow [\,(p \wedge q) \wedge r\,]$ | $[\, p \vee (q \vee r)\,] \Leftrightarrow [\,(p \vee q) \vee r\,]$ |
| De Morgan Kuralı | $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$ | $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ |
| Dağılma | $[\, p \wedge (q \vee r)\,] \Leftrightarrow [\,(p \wedge q) \vee (p \wedge r)\,]$ | $[\, p \vee (q \wedge r)\,] \Leftrightarrow [\,(p \vee q) \wedge (p \vee r)\,]$ |
| Eşdeğerlik | $(p \Leftrightarrow q) \Leftrightarrow [\,(p \Rightarrow q) \wedge (q \Rightarrow p)\,]$ | $(p \Leftrightarrow q) \Leftrightarrow [\,(p \wedge q) \vee (\neg p \wedge \neg q)\,]$ |
| Karşıtlık | $(p \Rightarrow q) \Leftrightarrow (\neg q \vee \neg p)$ | |
| İhraç etme | $[p \Rightarrow (q \Rightarrow r)] \Leftrightarrow [(p \wedge q) \Rightarrow r]$ | |
| Denkgüçlülük | $(p \wedge p) \Leftrightarrow p$ | $(p \vee p) \Leftrightarrow p$ |

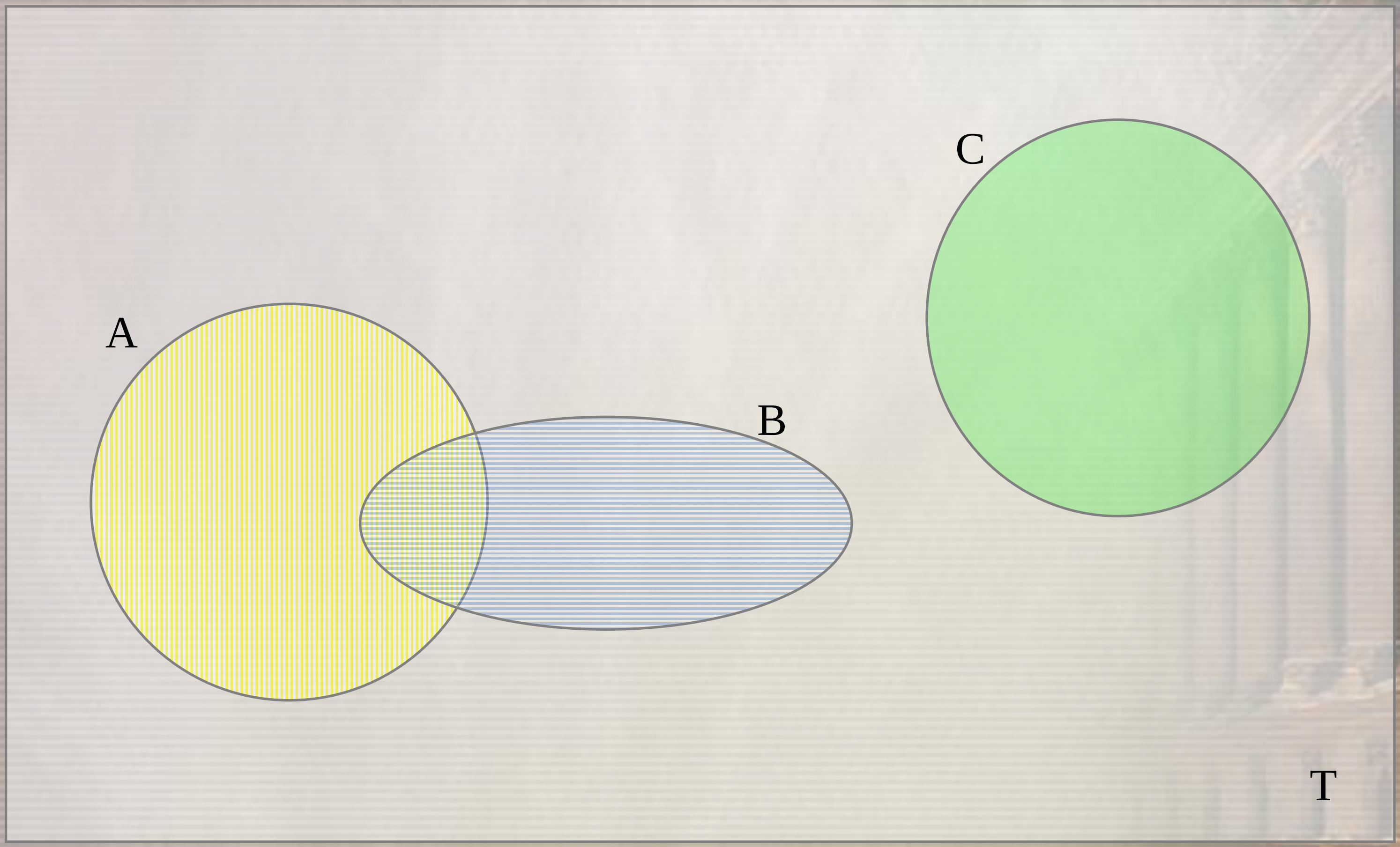
# Klasik Mantık

## eşdeğer gösterimler tablosu

| Kümeler Teorisi | Boolean Cebiri | Önermeler Mantığı |
|---|---|---|
| P(x) | B | P(v) |
| U | + | $\vee$ |
| $\cap$ | • | $\wedge$ |
| - | - | $\neg$ |
| x | 1 | 1 |
| ø | 0 | 0 |
| $\subseteq$ | ≤ | $\Rightarrow$ |

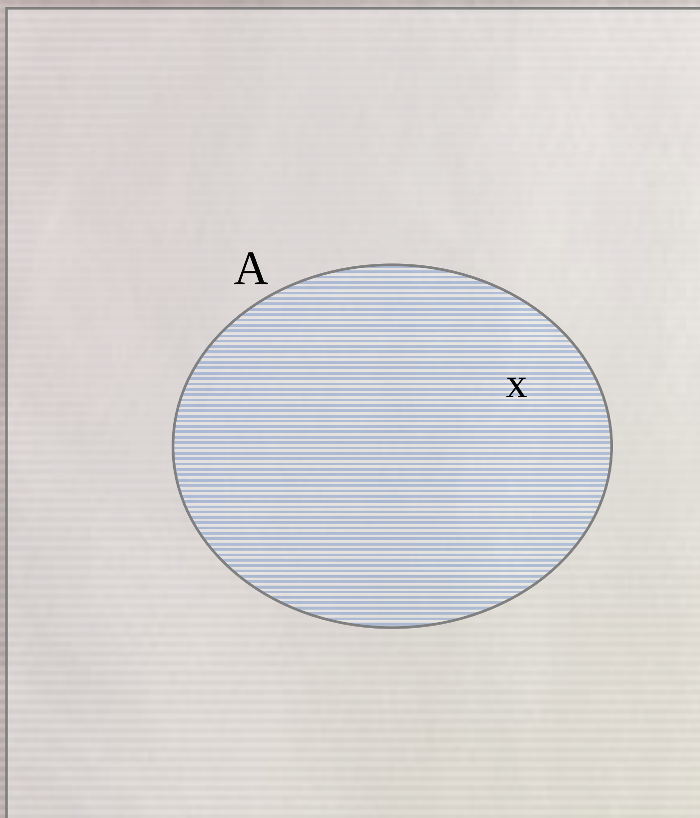
# Klasik Kümeler

venn diyagramı

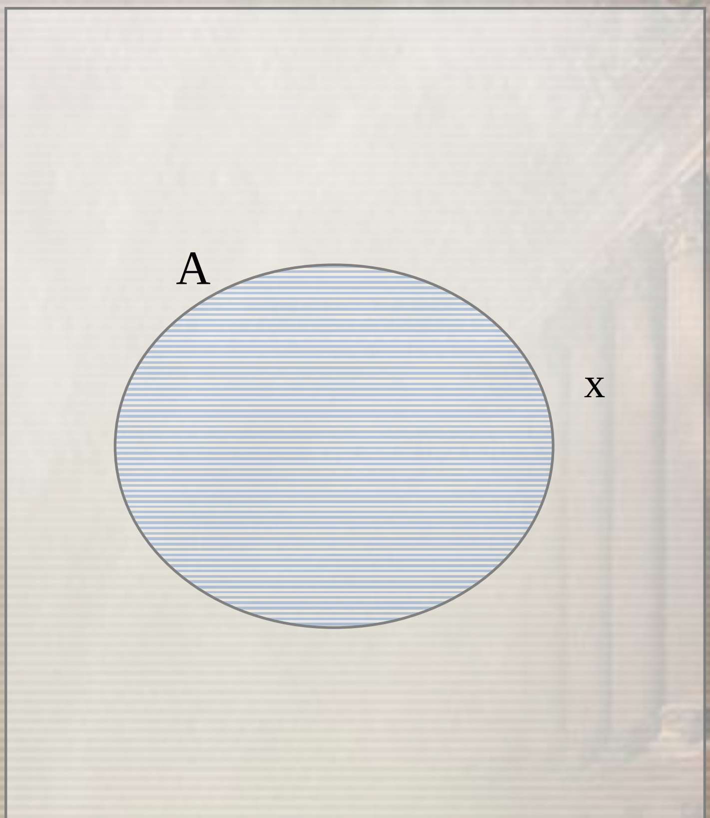


# Klasik Kümeler

eleman olma durumu



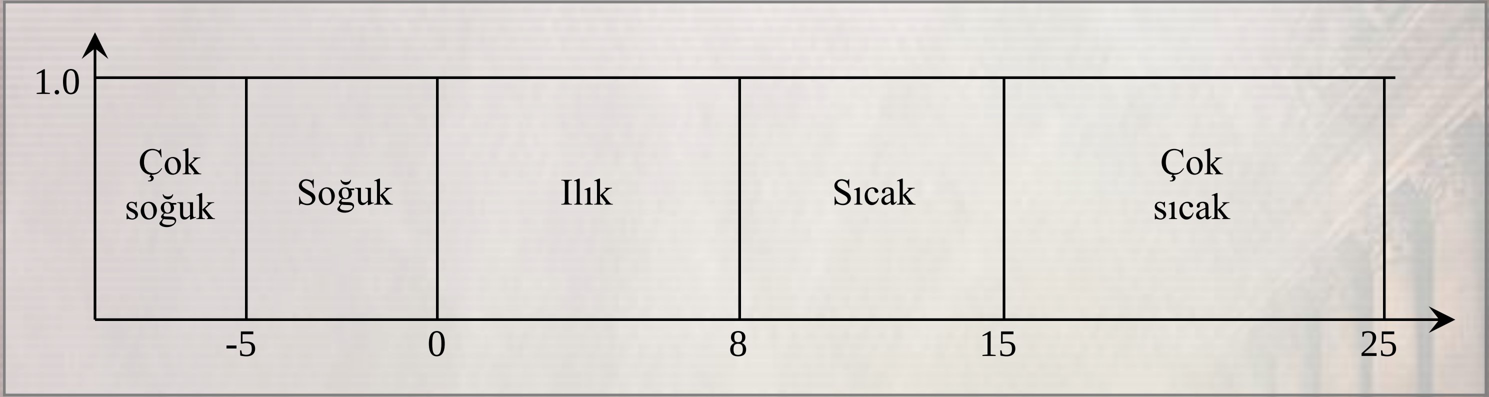

$x \in A$

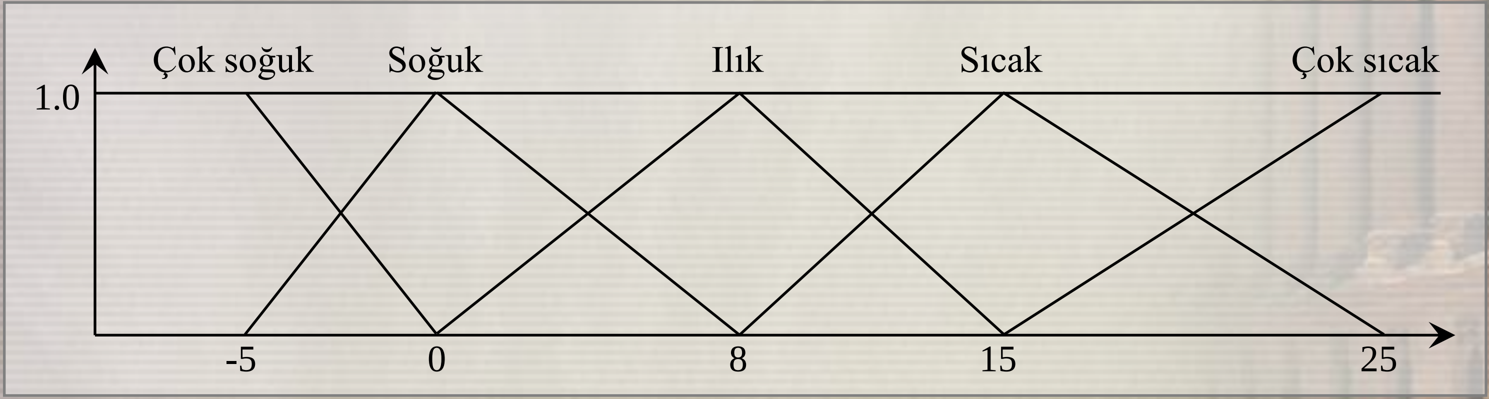



$x \notin A$

# Bulanık Kümeler

eleman olma durumu

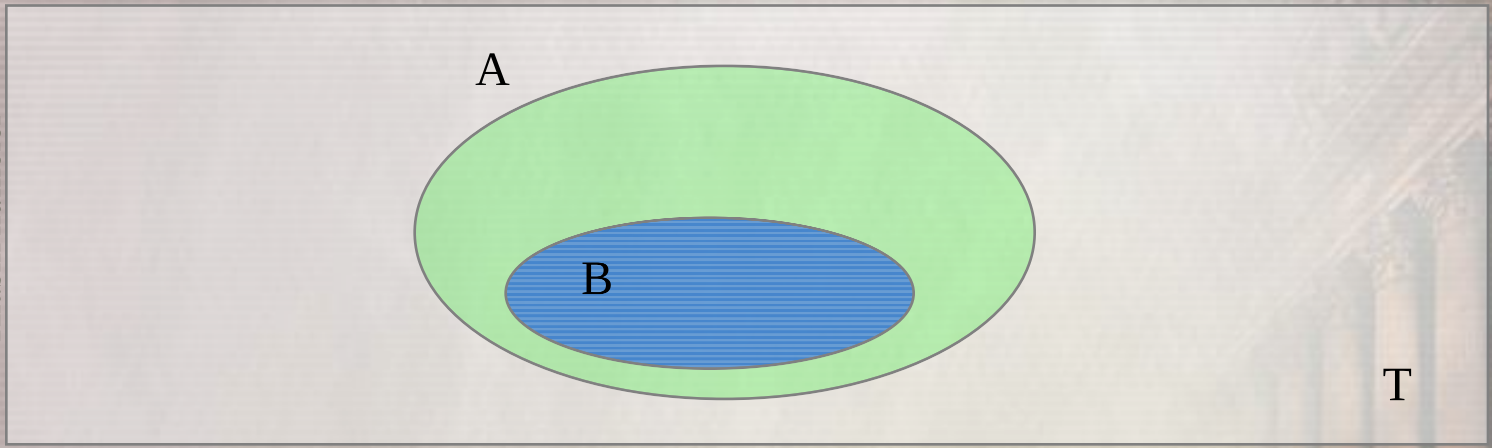


# Klasik ve Bulanık Kümeler

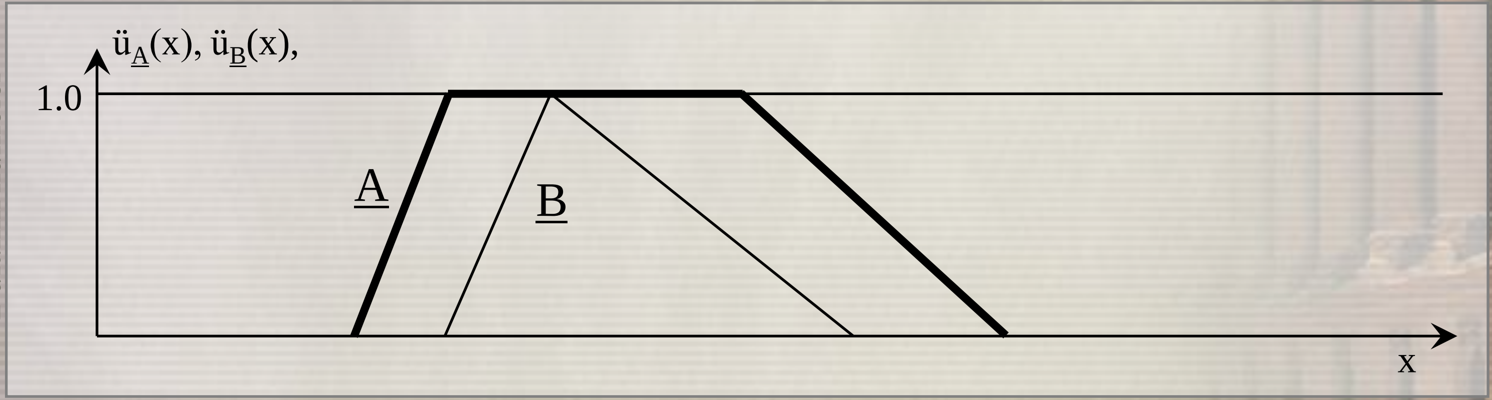
alt küme

Klasik kümeler




$A \subseteq B$ ve $\underline{A} \subseteq \underline{B}$

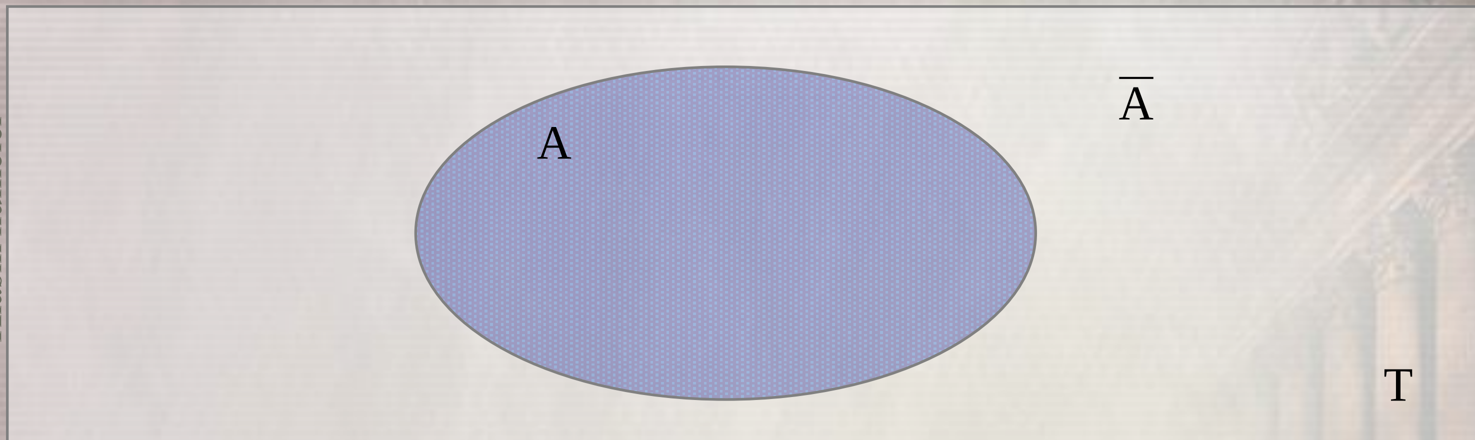
Bulanık kümeler



# Klasik ve Bulanık Kümeler

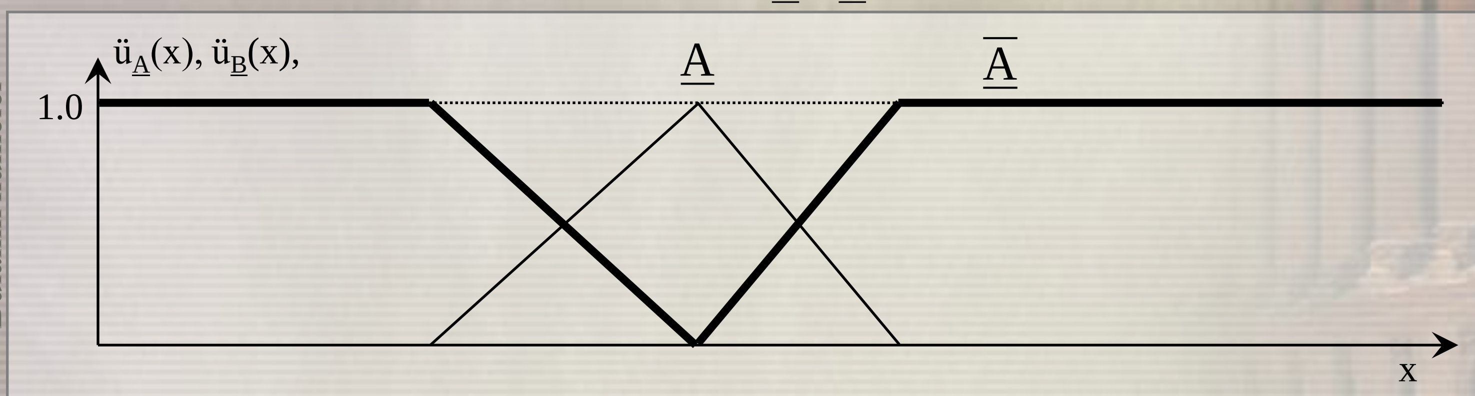
## tamamlayıcı kümeler

Klasik kümeler




$A \cup \overline{A} = T$ ve $\underline{A} \vee \overline{\underline{A}} = T$

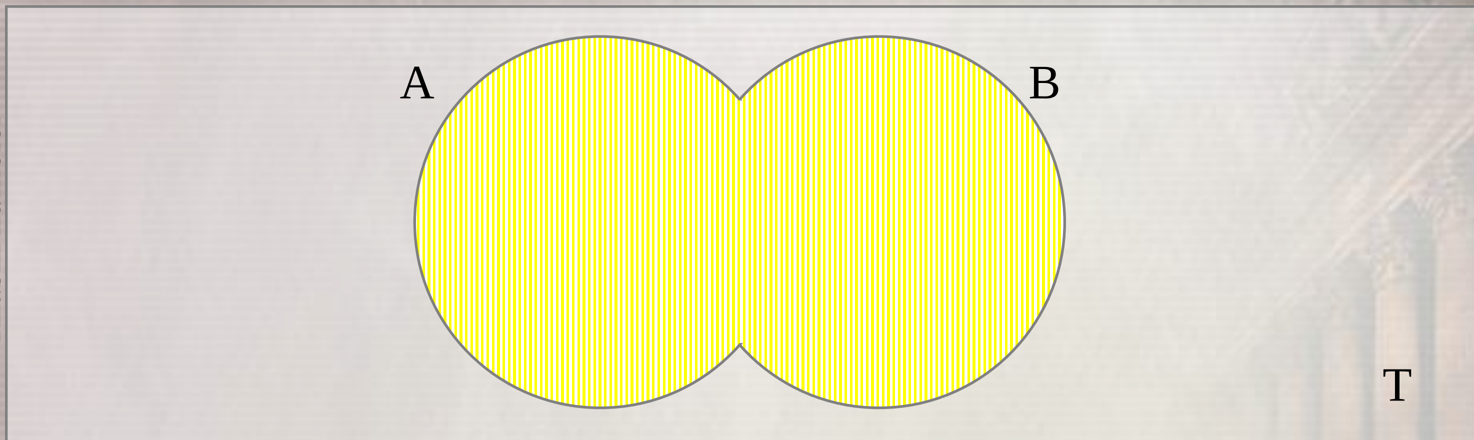
Bulanık kümeler



# Klasik ve Bulanık Kümeler

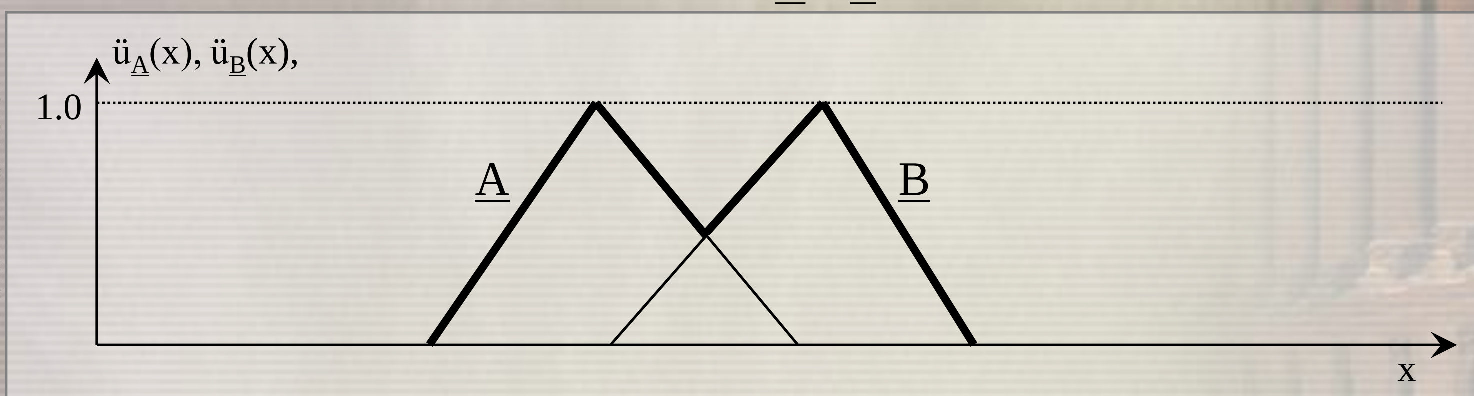
birleşim kümeleri

Klasik kümeler




$A \cup B$ ve $\underline{A} \vee \underline{B}$

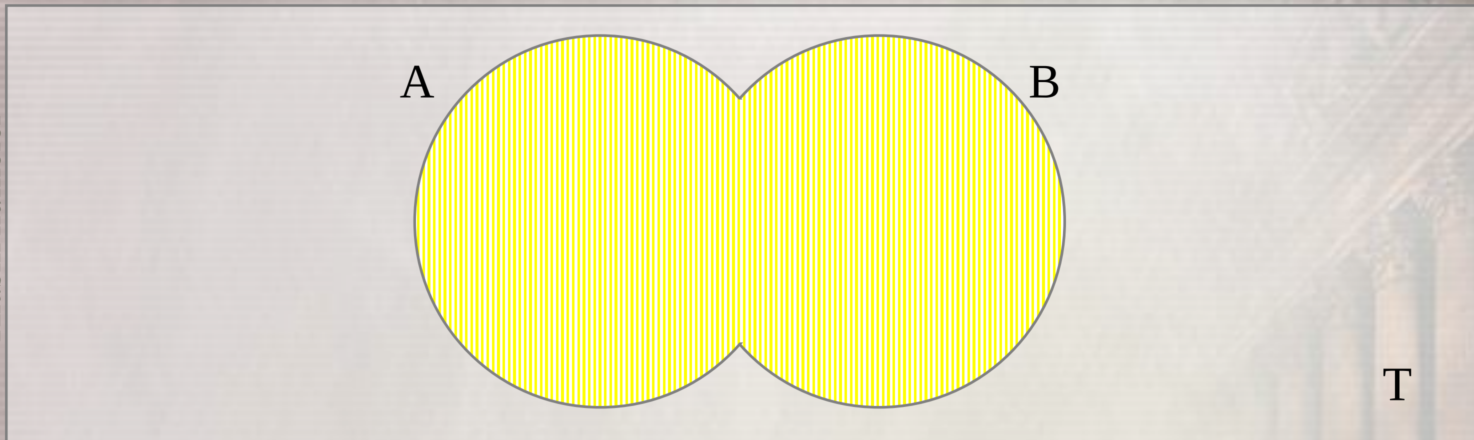
Bulanık kümeler



# Klasik ve Bulanık Kümeler

klasik kümelerde birleşim

Klasik kümeler




$A \cup B$

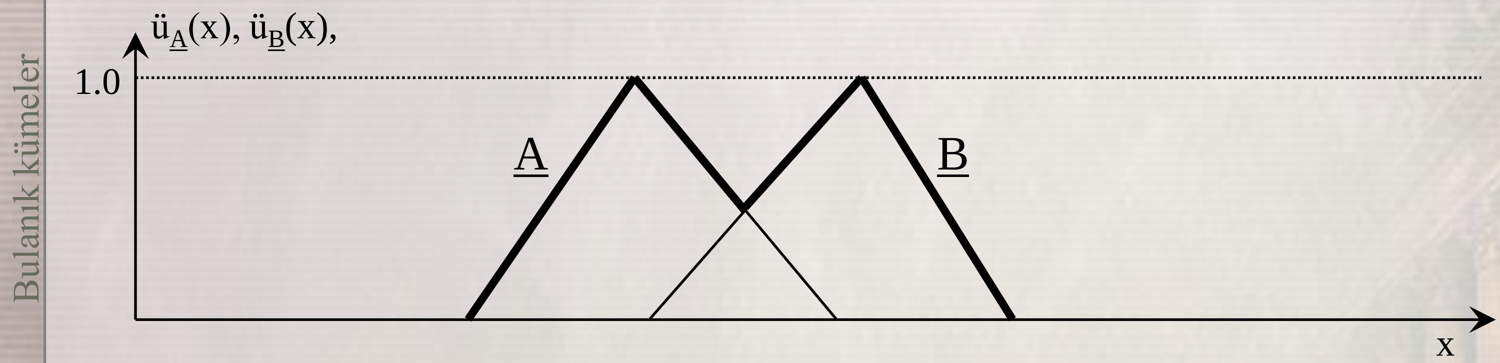
$A = \{a, b, c, e, h, i\}$

$B = \{1, a, 3, z, b, x\}$

$A \cup B = \{a, 1, 3, b, c, e, h, i, z, x\}$

# Klasik ve Bulanık Kümeler

## bulanık kümelerde birleşim




$\underline{A} \vee \underline{B}$

$$\underline{A} = \{0.1/a + 0.3/b + 0.9/c + 1.0/e + 0.6/h + 0.2/i\}$$

$$\underline{B} = \{1.0/1 + 0.8/a + 0.6/3 + 0.4/z + 0.2/b + 0.1/x\}$$

$$\underline{A} \vee \underline{B} = \{0.8/a + 1.0/1 + 0.6/3 + 0.3/b + 0.9/c + 1.0/e + 0.6/h + 0.2/i + 0.4/z + 0.1/x\}$$

---

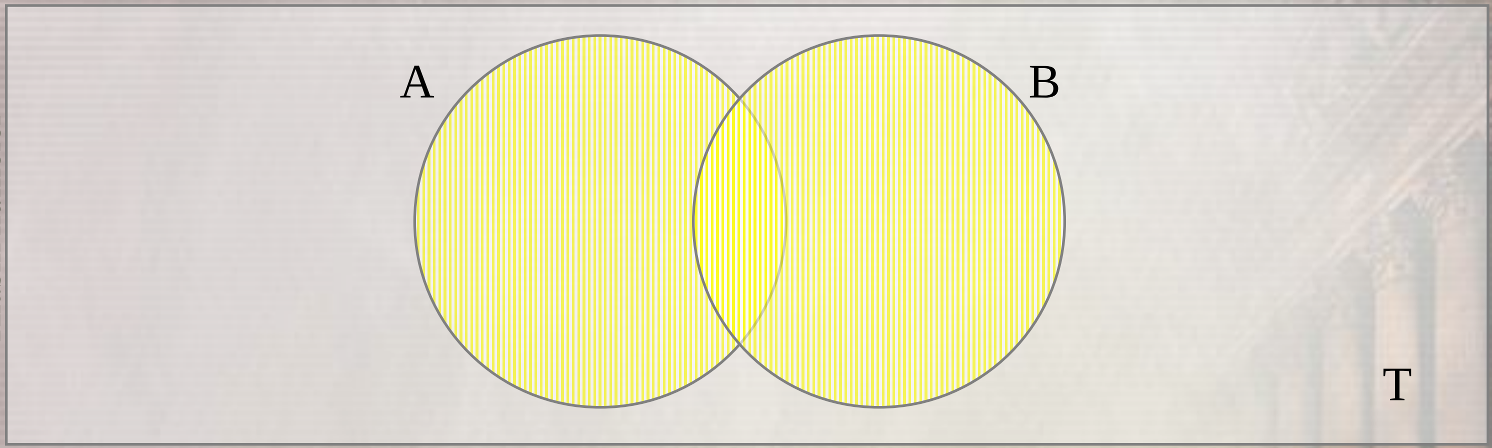
$$\text{ü}(a) = EB[0.1, 0.8] = 0.8$$

$$\text{ü}(b) = EB[0.3, 0.2] = 0.3$$

# Klasik ve Bulanık Kümeler

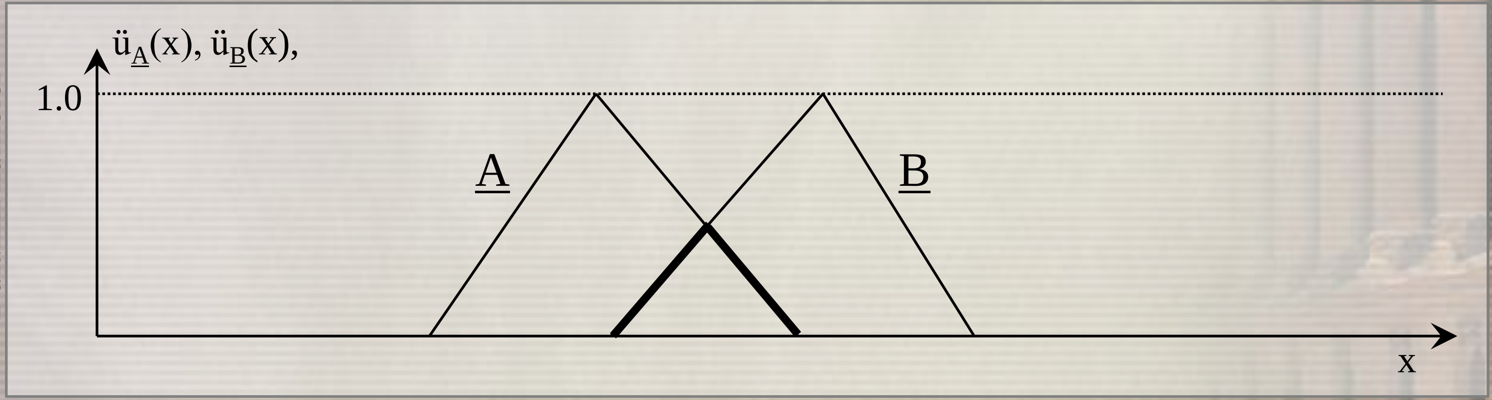
## kesişim kümeleri

Klasik kümeler




$A \cap B$ veya $\underline{A} \wedge \underline{B}$

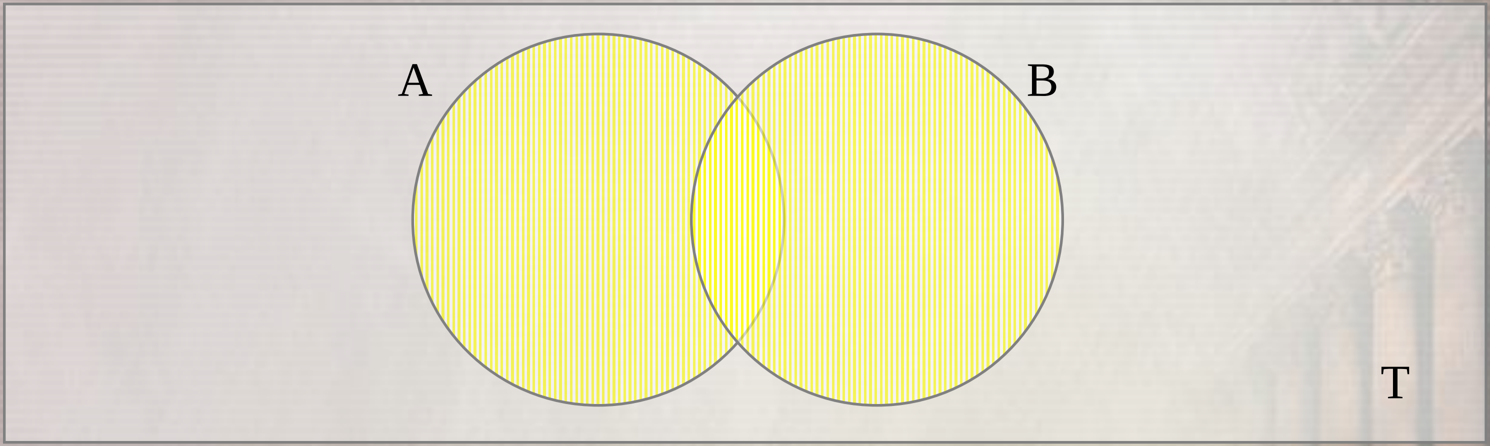
Bulanık kümeler



# Klasik ve Bulanık Kümeler

klasik kümelerde kesişim

Klasik kümeler




$A \cap B$

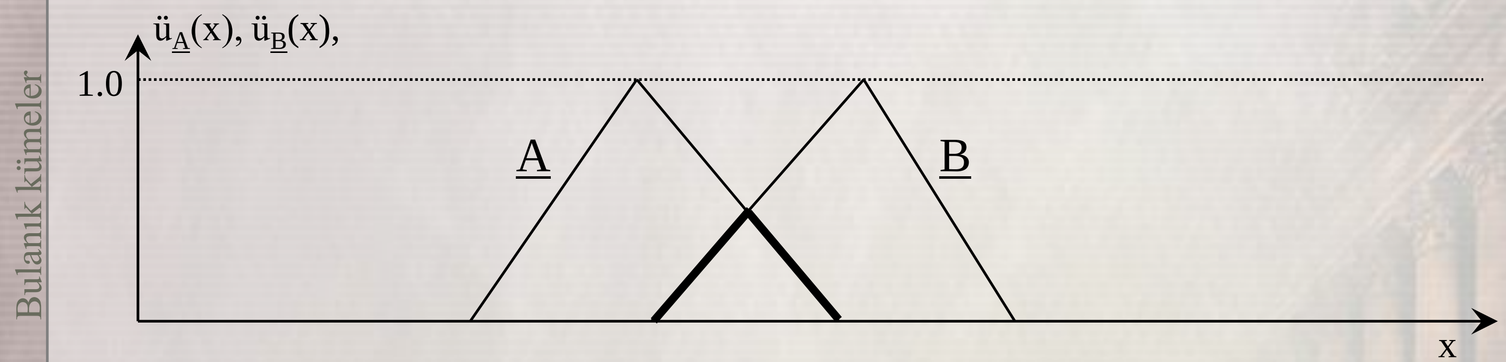
$A = \{a, b, c, e, h, i\}$

$B = \{1, a, 3, z, b, x\}$

$A \cap B = \{a, b\}$

# Klasik ve Bulanık Kümeler

## bulanık kümelerde kesişim




$\underline{A} \wedge \underline{B}$

$$\underline{A} = \{0.1/a + 0.3/b + 0.9/c + 1.0/e + 0.6/h + 0.2/i\}$$

$$\underline{B} = \{1.0/1 + 0.8/a + 0.6/3 + 0.4/z + 0.2/b + 0.1/x\}$$

$$\underline{A} \wedge \underline{B} = \{0.1/a + 0.2/b\}$$

---

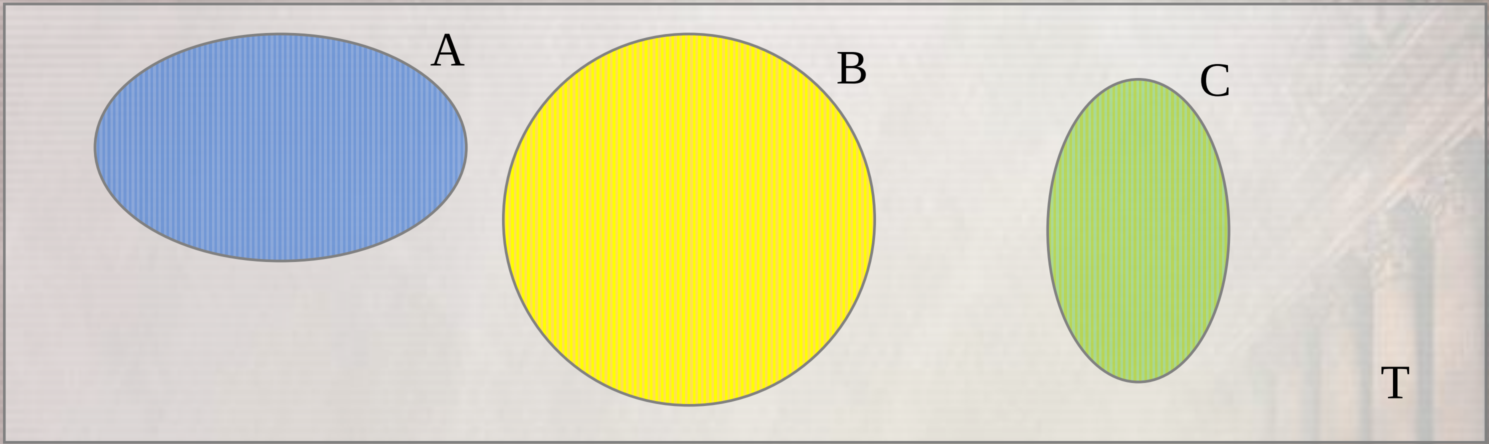
$$\ddot{u}(a) = \mathrm{EK}[0.1, 0.8] = 0.1$$

$$\ddot{u}(b) = \mathrm{EK}[0.3, 0.2] = 0.2$$

# Klasik ve Bulanık Kümeler

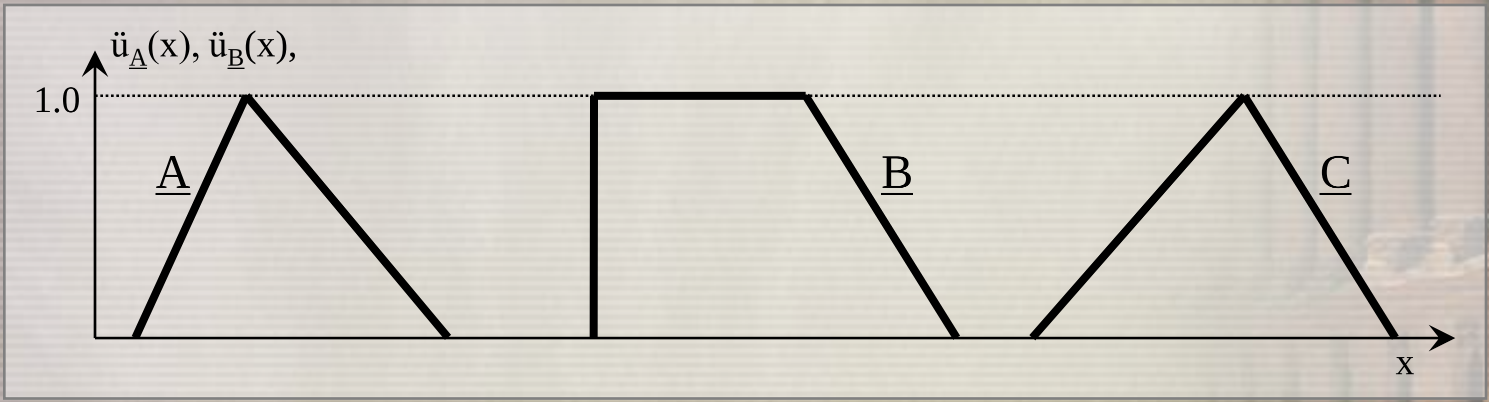
ayrık kümeler

Klasik kümeler



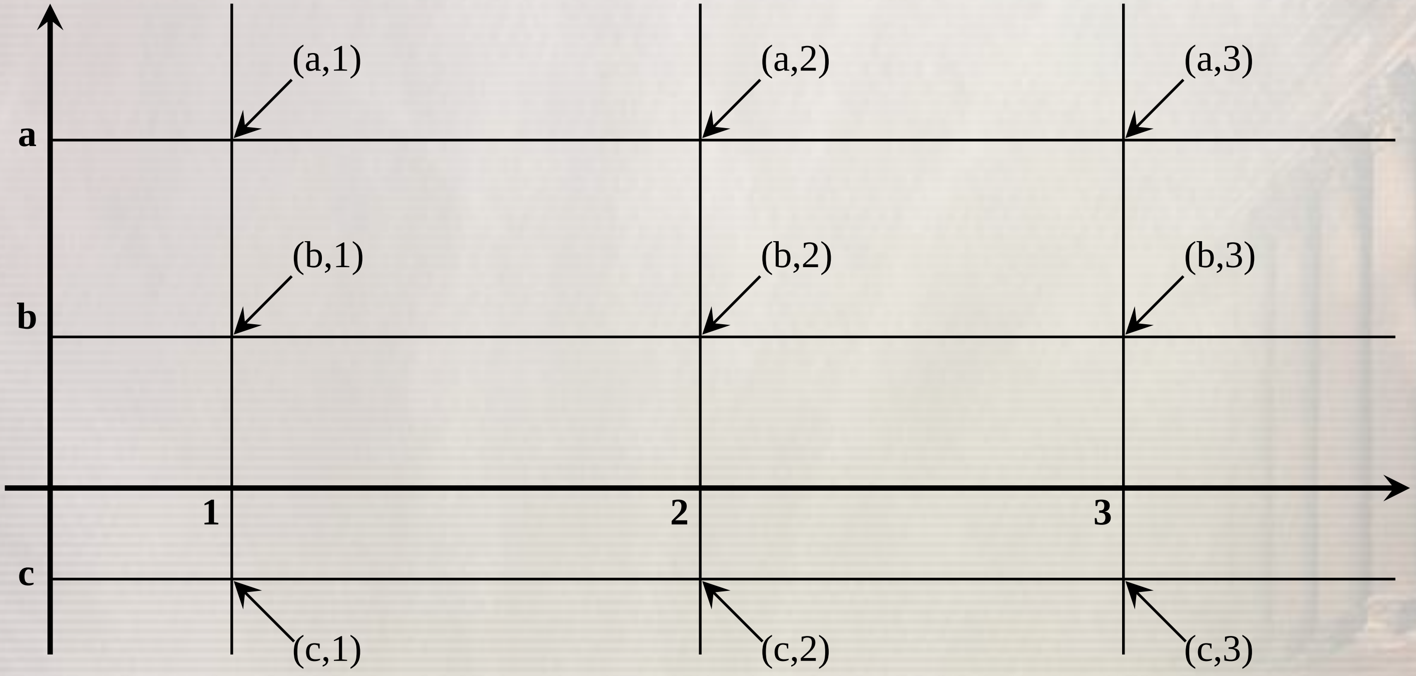

$A \cap B = \phi$ veya $\underline{A} \wedge \underline{B} = \phi$ $A \cap C = \phi$ veya $\underline{A} \wedge \underline{C} = \phi$ $B \cap C = \phi$ veya $\underline{B} \wedge \underline{C} = \phi$

Bulanık kümeler



# Klasik ve Bulanık Kümeler

çarpım kümesi



# Klasik ve Bulanık Kümeler

## çarpım kümeleri

Klasik kümeler

$A = \{a, b, c\}$

$B = \{1, 2, 3\}$

$C = A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$

Bulanık kümeler

$\underline{A} = \{0.1/a, 1.0/b, 0.3/c\}$

$\underline{B} = \{0.3/1 + 0.7/2 + 1.0/3\}$

$$\underline{A} \wedge \underline{B} = \{0.1/(1,a) + 0.3/(1,b) + 0.3/(1,c) + 0.1/(2,a) + 0.7/(2,b) + 0.3/(2,c)$$

$$+ 0.1/(3,a) + 1.0/(3,b) + 0.3/3,c)\}$$

---

$$ü_{\underline{A} \times \underline{B}}(x) = EK[ü_{\underline{A}}(x), ü_{\underline{B}}(x)]$$

# Klasik ve Bulanık Kümeler

## çoklu küme işlemleri

**Birleşme özelliği**

$$A \cup B = B \cup A = \underline{A} \vee \underline{B} = \underline{B} \vee \underline{A}$$

$$A \cap B = B \cap A = \underline{A} \wedge \underline{B} = \underline{B} \wedge \underline{A}$$

$$(A \cup B) \cup C = A \cup (B \cup C) = (\underline{A} \vee \underline{B}) \vee \underline{C} = \underline{A} \vee (\underline{B} \vee \underline{C})$$

$$(A \cap B) \cap C = A \cap (B \cap C) = (\underline{A} \wedge \underline{B}) \wedge \underline{C} = \underline{A} \wedge (\underline{B} \wedge \underline{C})$$

**Dağılma özelliği**

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \underline{A} \wedge (\underline{B} \vee \underline{C}) = (\underline{A} \wedge \underline{B}) \vee (\underline{A} \wedge \underline{C})$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \underline{A} \vee (\underline{B} \wedge \underline{C}) = (\underline{A} \vee \underline{B}) \wedge (\underline{A} \vee \underline{C})$$

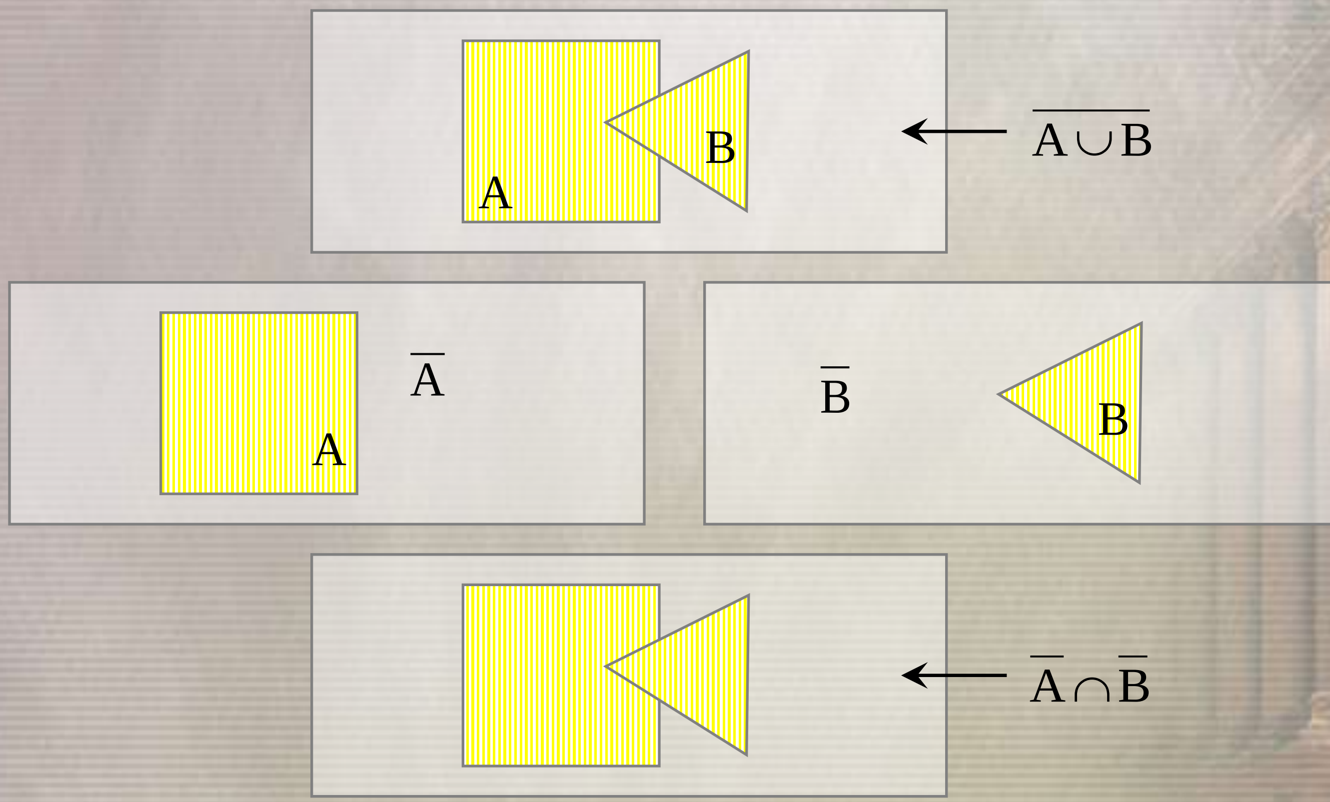
**De Morgan kuralı**

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

# Klasik ve Bulanık Kümeler

De Morgan kuralı



# Teşekkürler