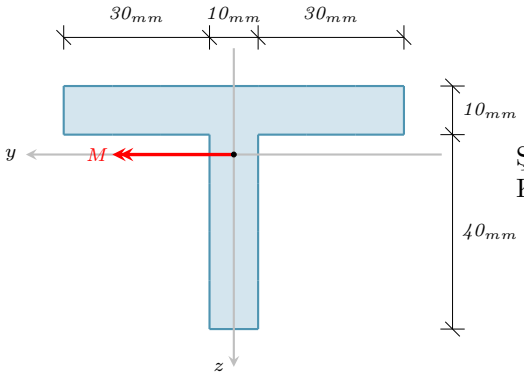
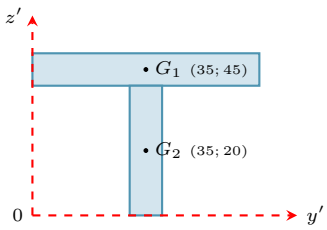
8.1 Örnek



Şekildeki kesite 1.5 kNm şiddetinde bir moment etkimektedir. Kesitte oluşan en büyük normal gerilmeleri hesaplayınız.

Kesit ağırlık merkezi :



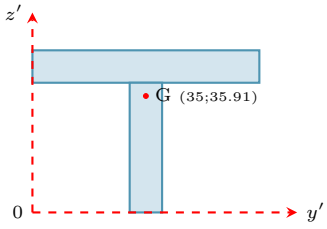
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$) :

$$G_{y'} = \frac{A_1 \cdot y_1' + A_2 \cdot y_2'}{A_1 + A_2}$$

$$G_{y'} = \frac{700 \cdot 35 + 400 \cdot 35}{700 + 400}$$

$$G_{y'} = \frac{38500}{1100}$$

$$G_{y'} = 35 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$) :

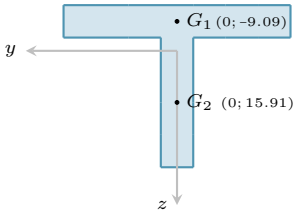
$$G_{z'} = \frac{A_1 \cdot z_1' + A_2 \cdot z_2'}{A_1 + A_2}$$

$$G_{z'} = \frac{700 \cdot 45 + 400 \cdot 20}{700 + 400}$$

$$G_{z'} = \frac{39500}{1100}$$

$$G_{z'} = 35.91 \text{ mm}$$

a. I_y, I_z ve I_{yz} atalet momentleri :



Ağırlık merkezlerinin eksenlere olan uzaklıkları :

$G_1\,(0;\,-9.09) \Rightarrow \quad y_1 = 0$ (mm) $\quad z_1 =$ -9.09 (mm)

$G_2\,(0;\,15.91) \Rightarrow \quad y_2 = 0$ (mm) $\quad z_2 = 15.91$ (mm)

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot {z_1}^2 = \frac{70 \cdot 10^3}{12} + 700 \cdot (-9.09)^2 \quad \Rightarrow \quad I_y^1 = 63673\ mm^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot {z_2}^2 = \frac{10 \cdot 40^3}{12} + 400 \cdot (15.91)^2 \quad \Rightarrow \quad I_y^2 = 154584.56\ mm^4$$

$$I_y = I_y^1 + I_y^2$$

$$I_y = 63673 + 154584.56 \quad \Rightarrow \quad I_y = 218258\ mm^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot {y_1}^2 = \frac{10 \cdot 70^3}{12} + 700 \cdot (0)^2 \quad \Rightarrow \quad I_z^1 = 285833.34\ mm^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot {y_2}^2 = \frac{40 \cdot 10^3}{12} + 400 \cdot (0)^2 \quad \Rightarrow \quad I_z^2 = 3333.33\ mm^4$$

$$I_z = I_z^1 + I_z^2$$

$$I_z = 285833.34 + 3333.33 \quad \Rightarrow \quad I_z = 289167\ mm^4$$

Çarpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 700 \cdot 0 \cdot 9.09 \quad \Rightarrow \quad I_{yz}^1 = 0\ mm^4$$

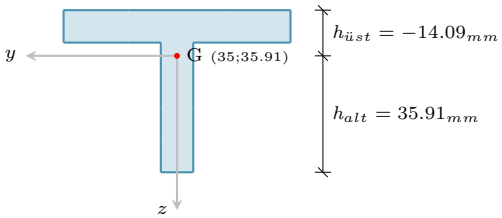
$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 400 \cdot 0 \cdot (-15.91) \quad \Rightarrow \quad I_{yz}^2 = 0\ mm^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2$$

$$I_{yz} = 0 + 0 \quad \Rightarrow \quad I_{yz} = 0\ mm^4$$

Çarpım atalet momenti $I_{yz} = 0$ olduğundan yz eksen takımı, aynı zamanda asal eksen takımıdır. Ayrıca kesite etkiyen eğilme momenti, y eksenine paralel olduğundan, y ekseni t arafsız eksendir ve bu problem bir ***düz eğilme problemi*** *olarak çözülebilir.*

Mukavemet momentleri ($W_{alt,üst}$) :



$$W_{alt} = \frac{I_y}{h_{alt}} = \frac{218258}{35.91} \Rightarrow W_{alt} = 6077.92\ mm^3$$

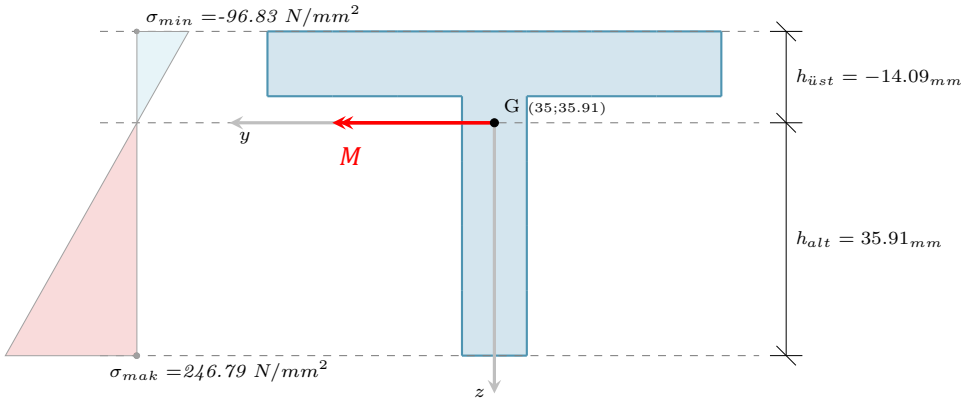
$$W_{üst} = \frac{I_y}{h_{üst}} = \frac{218258}{-14.09} \Rightarrow W_{üst} = -15490.28\ mm^3$$

Gerilme değerleri ($\sigma_{alt,üst}$) :

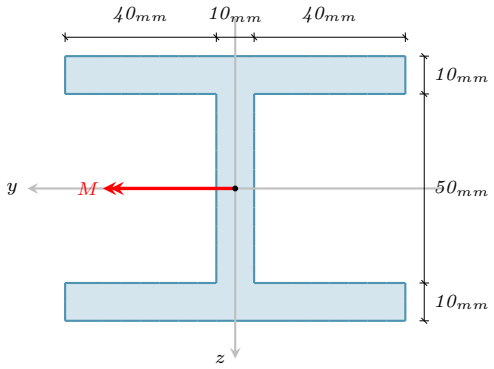
$$\sigma_{alt} = \frac{M_y}{W_{alt}} = \frac{1500000}{6077.92} \Rightarrow \sigma_{alt} = 246.79\ N/mm^2$$

$$\sigma_{üst} = \frac{M_y}{W_{üst}} = \frac{1500000}{-15490.28} \Rightarrow \sigma_{üst} = -96.83\ N/mm^2$$

Gerilme dağılımı :

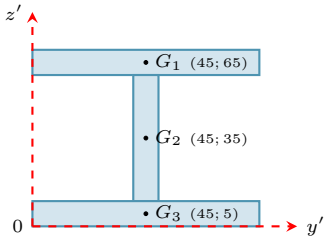


8.2 Örnek



Şekildeki kesite 2.5 kNm şiddetinde bir moment etkimektedir.
Kesitte oluşan en büyük ve en küçük normal gerilmeleri hesaplayınız.

Kesit ağırlık merkezi :



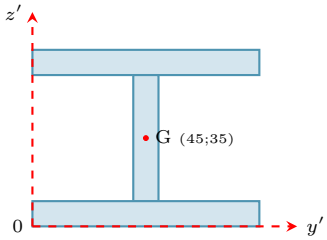
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$) :

$$G_{y'} = \frac{A_1 \cdot y'_1 + A_2 \cdot y'_2 + A_3 \cdot y'_3}{A_1 + A_2 + A_3}$$

$$G_{y'} = \frac{900 \cdot 45 + 500 \cdot 45 + 900 \cdot 45}{900 + 500 + 900}$$

$$G_{y'} = \frac{103500}{2300}$$

$$G_{y'} = 45 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$) :

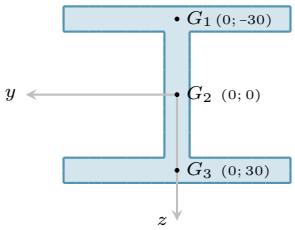
$$G_{z'} = \frac{A_1 \cdot z'_1 + A_2 \cdot z'_2 + A_3 \cdot z'_3}{A_1 + A_2 + A_3}$$

$$G_{z'} = \frac{900 \cdot 65 + 500 \cdot 35 + 900 \cdot 5}{900 + 500 + 900}$$

$$G_{z'} = \frac{80500}{2300}$$

$$G_{z'} = 35 \text{ mm}$$

a. I_y, I_z ve I_{yz} atalet momentleri :



Ağırlık merkezlerinin eksenlere olan uzaklıkları :

$G_1\,(0;-30) \quad \Rightarrow \quad y_1 = 0\ (\text{mm}) \quad z_1 = -30\ (\text{mm})$

$G_2\ (0;0) \quad \Rightarrow \quad y_2 = 0\ (\text{mm}) \quad z_2 = 0\ (\text{mm})$

$G_3\ (0;30) \quad \Rightarrow \quad y_3 = 0\ (\text{mm}) \quad z_3 = 30\ (\text{mm})$

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot {z_1}^2 = \frac{90 \cdot 10^3}{12} + 900 \cdot (-30)^2 \quad \Rightarrow \quad I_y^1 = 817500\ mm^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot {z_2}^2 = \frac{10 \cdot 50^3}{12} + 500 \cdot (0)^2 \quad \Rightarrow \quad I_y^2 = 104166.67\ mm^4$$

$$I_y^3 = \frac{b_3 \cdot h_3^3}{12} + A_3 \cdot {z_3}^2 = \frac{90 \cdot 10^3}{12} + 900 \cdot (30)^2 \quad \Rightarrow \quad I_y^3 = 817500\ mm^4$$

$$I_y = I_y^1 + I_y^2 + I_y^3$$

$$I_y = 817500 + 104166.67 + 817500 \quad \Rightarrow \quad I_y = 1739167\ mm^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot {y_1}^2 = \frac{10 \cdot 90^3}{12} + 900 \cdot (0)^2 \quad \Rightarrow \quad I_z^1 = 607500\ mm^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot {y_2}^2 = \frac{50 \cdot 10^3}{12} + 500 \cdot (0)^2 \quad \Rightarrow \quad I_z^2 = 4166.67\ mm^4$$

$$I_z^3 = \frac{h_3 \cdot b_3^3}{12} + A_3 \cdot {y_3}^2 = \frac{10 \cdot 90^3}{12} + 900 \cdot (0)^2 \quad \Rightarrow \quad I_z^3 = 607500\ mm^4$$

$$I_z = I_z^1 + I_z^2 + I_z^3$$

$$I_z = 607500 + 4166.67 + 607500 \quad \Rightarrow \quad I_z = 1219167\ mm^4$$

Çarpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 900 \cdot 0 \cdot 30 \quad \Rightarrow \quad I_{yz}^1 = 0\ mm^4$$

$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 500 \cdot 0 \cdot 0 \quad \Rightarrow \quad I_{yz}^2 = 0\ mm^4$$

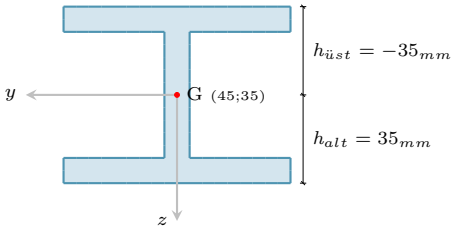
$$I_{yz}^3 = A_3 \cdot y_3 \cdot z_3 = 900 \cdot 0 \cdot (-30) \quad \Rightarrow \quad I_{yz}^3 = 0\ mm^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2 + I_{yz}^3$$

$$I_{yz} = 0 + 0 + 0 \quad \Rightarrow \quad I_{yz} = 0\ mm^4$$

*Çarpım atalet momenti $I_{yz} = 0$ olduğundan yz eksen takımı, aynı zamanda asal eksen takımıdır. Ayrıca kesite etkiyen eğilme momenti, y eksenine paralel olduğundan, y ekseni t arafsız eksendir ve bu problem bir **düz eğilme problemi** olarak çözülebilir.*

Mukavemet momentleri ($W_{alt,üst}$) :



$$W_{alt} = \frac{I_y}{h_{alt}} = \frac{1739167}{35} \quad \Rightarrow \quad W_{alt} = 49690.48\ mm^3$$

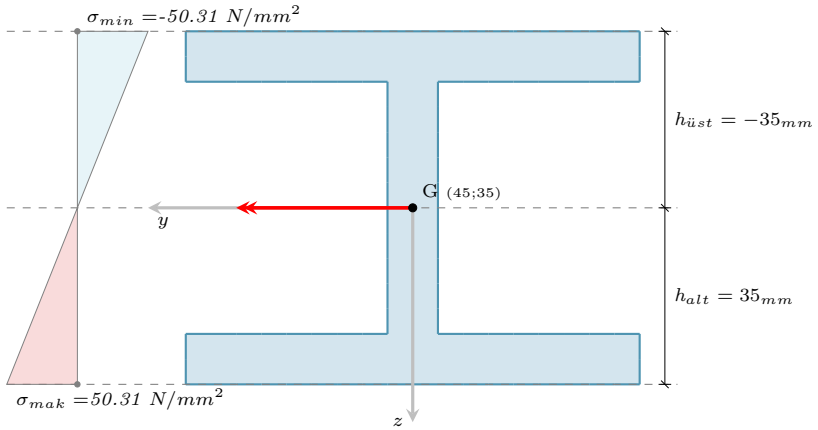
$$W_{üst} = \frac{I_y}{h_{üst}} = \frac{1739167}{-35} \quad \Rightarrow \quad W_{üst} = -49690.48\ mm^3$$

Gerilme değerleri ($\sigma_{alt,üst}$) :

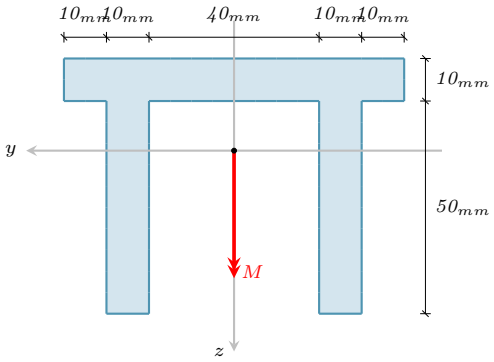
$$\sigma_{alt} = \frac{M_y}{W_{alt}} = \frac{2500000}{49690.48} \quad \Rightarrow \quad \sigma_{alt} = 50.31\ N/mm^2$$

$$\sigma_{üst} = \frac{M_y}{W_{üst}} = \frac{2500000}{-49690.48} \quad \Rightarrow \quad \sigma_{üst} = -50.31\ N/mm^2$$

Gerilme dağılımı :

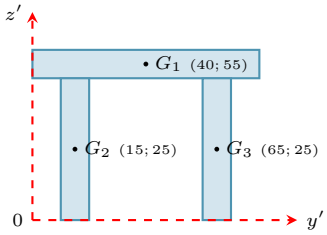


8.3 Örnek



Şekildeki kesite 2.5 kNm şiddetinde bir moment etkimektedir.
Kesitte oluşan en büyük ve en küçük normal gerilmeleri hesaplayınız.

Kesit ağırlık merkezi :



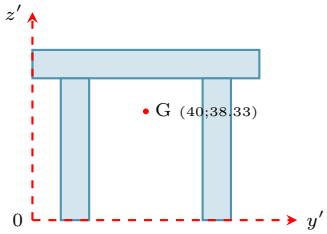
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$) :

$$G_{y'} = \frac{A_1 \cdot y'_1 + A_2 \cdot y'_2 + A_3 \cdot y'_3}{A_1 + A_2 + A_3}$$

$$G_{y'} = \frac{800 \cdot 40 + 500 \cdot 15 + 500 \cdot 65}{800 + 500 + 500}$$

$$G_{y'} = \frac{72000}{1800}$$

$$G_{y'} = 40 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$) :

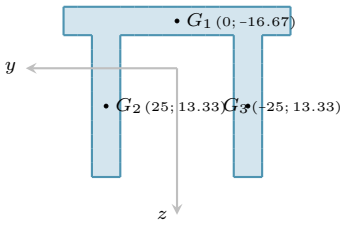
$$G_{z'} = \frac{A_1 \cdot z'_1 + A_2 \cdot z'_2 + A_3 \cdot z'_3}{A_1 + A_2 + A_3}$$

$$G_{z'} = \frac{800 \cdot 55 + 500 \cdot 25 + 500 \cdot 25}{800 + 500 + 500}$$

$$G_{z'} = \frac{69000}{1800}$$

$$G_{z'} = 38.33 \text{ mm}$$

a. I_y, I_z ve I_{yz} atalet momentleri :



Ağırlık merkezlerinin eksenlere olan uzaklıkları :

G_1 (0; –16.67) $\Rightarrow$ $y_1 = 0$ (mm) z_1 = -16.67 (mm)

G_2 (25; 13.33 $\Rightarrow$ $y_2 = 25$ (mm) $z_2 = 13.33$ (mm)

G_3 (–25; 13.33 $\Rightarrow$ y_3 = -25 (mm) $z_3 = 13.33$ (mm)

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot {z_1}^2 = \frac{80 \cdot 10^3}{12} + 800 \cdot (-16.67)^2 \quad \Rightarrow \quad I_y^1 = 228977.8 \; mm^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot {z_2}^2 = \frac{10 \cdot 50^3}{12} + 500 \cdot (13.33)^2 \quad \Rightarrow \quad I_y^2 = 193011.12 \; mm^4$$

$$I_y^3 = \frac{b_3 \cdot h_3^3}{12} + A_3 \cdot {z_3}^2 = \frac{10 \cdot 50^3}{12} + 500 \cdot (13.33)^2 \quad \Rightarrow \quad I_y^3 = 193011.12 \; mm^4$$

$$I_y = I_y^1 + I_y^2 + I_y^3$$

$$I_y = 228977.8 + 193011.12 + 193011.12 \quad \Rightarrow \quad I_y = 615000 \; mm^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot {y_1}^2 = \frac{10 \cdot 80^3}{12} + 800 \cdot (0)^2 \quad \Rightarrow \quad I_z^1 = 426666.66 \; mm^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot {y_2}^2 = \frac{50 \cdot 10^3}{12} + 500 \cdot (-25)^2 \quad \Rightarrow \quad I_z^2 = 316666.66 \; mm^4$$

$$I_z^3 = \frac{h_3 \cdot b_3^3}{12} + A_3 \cdot {y_3}^2 = \frac{50 \cdot 10^3}{12} + 500 \cdot (25)^2 \quad \Rightarrow \quad I_z^3 = 316666.66 \; mm^4$$

$$I_z = I_z^1 + I_z^2 + I_z^3$$

$$I_z = 426666.66 + 316666.66 + 316666.66 \quad \Rightarrow \quad I_z = 1060000 \; mm^4$$

Çarpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 800 \cdot 0 \cdot 16.67 \quad \Rightarrow \quad I_{yz}^1 = 0\ mm^4$$

$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 500 \cdot (-25) \cdot (-13.33) \quad \Rightarrow \quad I_{yz}^2 = 166250\ mm^4$$

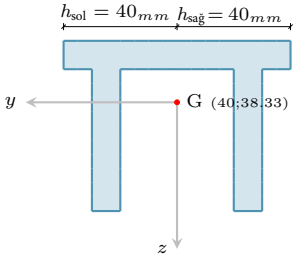
$$I_{yz}^3 = A_3 \cdot y_3 \cdot z_3 = 500 \cdot 25 \cdot (-13.33) \quad \Rightarrow \quad I_{yz}^3 = -166250\ mm^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2 + I_{yz}^3$$

$$I_{yz} = 0 + 166250 - 166250 \quad \Rightarrow \quad I_{yz} = 0\ mm^4$$

*Çarpım atalet momenti $I_{yz} = 0$ olduğundan yz eksen takımı, aynı zamanda asal eksen takımıdır. Ayrıca kesite etkiyen eğilme momenti, z eksenine paralel olduğundan, z ekseni t arafsız eksendir ve bu problem bir **düz eğilme problemi** olarak çözülebilir.*

Mukavemet momentleri ($W_{sol,sağ}$) :



$$W_{sol} = \frac{I_z}{h_{sol}} = \frac{1060000}{40} \quad \Rightarrow \quad W_{sol} = 26500\ mm^3$$

$$W_{sağ} = \frac{I_z}{h_{sağ}} = \frac{1060000}{-40} \quad \Rightarrow \quad W_{sağ} = -26500\ mm^3$$

Gerilme değerleri ($\sigma_{sol,sağ}$) :

$$\sigma_{sol} = -\frac{M_z}{W_{sol}} = -\frac{2500000}{26500} \quad \Rightarrow \quad \sigma_{sol} = -94.34\ N/mm^2$$

$$\sigma_{sağ} = -\frac{M_z}{W_{sağ}} = -\frac{2500000}{-26500} \quad \Rightarrow \quad \sigma_{sağ} = 94.34\ N/mm^2$$

Gerilme dağılımı :