

# Klasik Mantık ve Bulanık Mantığa Giriş

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İstanbul Üniversitesi İnşaat Mühendisliği Bölümü

# Mantıksal Bağlar

işaretler

<i>İşaret</i>	<i>Anlamı</i>
$\neg$	değil
$\wedge$	ve
$\vee$	veya
$\Rightarrow$	ise
$\Leftrightarrow$	sadece ve sadece

# Mantıksal Bağlar

tersini alma

p	$\neg p$
D	Y
Y	D

p	$\neg p$
1	0
0	1

$$|\neg p| = 1 - |p|$$

$|p|$  : p önermesinin gerçek değeri

# Mantıksal Bağlar

kesişim

p	q	$p \wedge q$
D	D	D
D	Y	Y
Y	D	Y
Y	Y	Y

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

$$|p \wedge q| = \min[|p|, |q|]$$

$$|p \wedge q| = |p| \cdot |q|$$

$$|p \wedge q| = \max[0, |p| + |q| - 1]$$

# Mantıksal Bağlar

birleşim

p	q	$p \vee q$
D	D	D
D	Y	D
Y	D	D
Y	Y	Y

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

$$|p \vee q| = \max [ |p|, |q| ]$$

$$|p \vee q| = \min [ 1, |p| + |q| ]$$

# Mantıksal Bağlar

koşullu önermeler

p	q	$p \Rightarrow q$
D	D	D
D	Y	Y
Y	D	D
Y	Y	D

p	q	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

$$|p \Rightarrow q| = \min [ 1, 1 + |q| - |p| ]$$

$$|p \Rightarrow q| = 1 - |p|(1 - |q|)$$

$$|p \Rightarrow q| \neq |q \Rightarrow p|$$

# Mantıksal Bağlar

çift yönlü koşullu önermeler

p	q	$p \Leftrightarrow q$
D	D	D
D	Y	Y
Y	D	Y
Y	Y	D

p	q	$p \Leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

$$|p \Leftrightarrow q| = |p| \cdot |q| + |\neg p| \cdot |\neg q|$$

# Mantıksal Bağlar

çelişki

**Çelişki:** Basit önermelerinin doğruluk değeri ne olursa olsun yanlış olan bileşke önermedir

$p$	$p \wedge \neg p$
D	Y
Y	Y

$p$	$p \wedge \neg p$
1	0
0	0

# Mantıksal Bağlar

totoloji

**Totoloji:** Basit önermelerinin doğruluk değeri ne olursa olsun doğru olan bileşke önermedir

$p$	$p \vee \neg p$
D	D
Y	D

$p$	$p \vee \neg p$
1	1
0	1

# Klasik Mantık

## yer deęiřtirme kuralları

Çift deęilleme	$p \Leftrightarrow \neg\neg p$	
Deęiřme	$(p \wedge q) \Leftrightarrow (q \wedge p)$	$(p \vee q) \Leftrightarrow (q \vee p)$
Birleřme	$[p \wedge (q \wedge r)] \Leftrightarrow [(p \wedge q) \wedge r]$	$[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$
De Morgan Kuralı	$\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$	$\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$
Daęılma	$[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$	$[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$
Eřdeęerlik	$(p \Leftrightarrow q) \Leftrightarrow [(p \Rightarrow q) \wedge (q \Rightarrow p)]$	$(p \Leftrightarrow q) \Leftrightarrow [(p \wedge q) \vee (\neg p \wedge \neg q)]$
Karřıtlık	$(p \Rightarrow q) \Leftrightarrow (\neg q \vee \neg p)$	
İhraç etme	$[p \Rightarrow (q \Rightarrow r)] \Leftrightarrow [(p \wedge q) \Rightarrow r]$	
Denkgüçlölük	$(p \wedge p) \Leftrightarrow p$	$(p \vee p) \Leftrightarrow p$

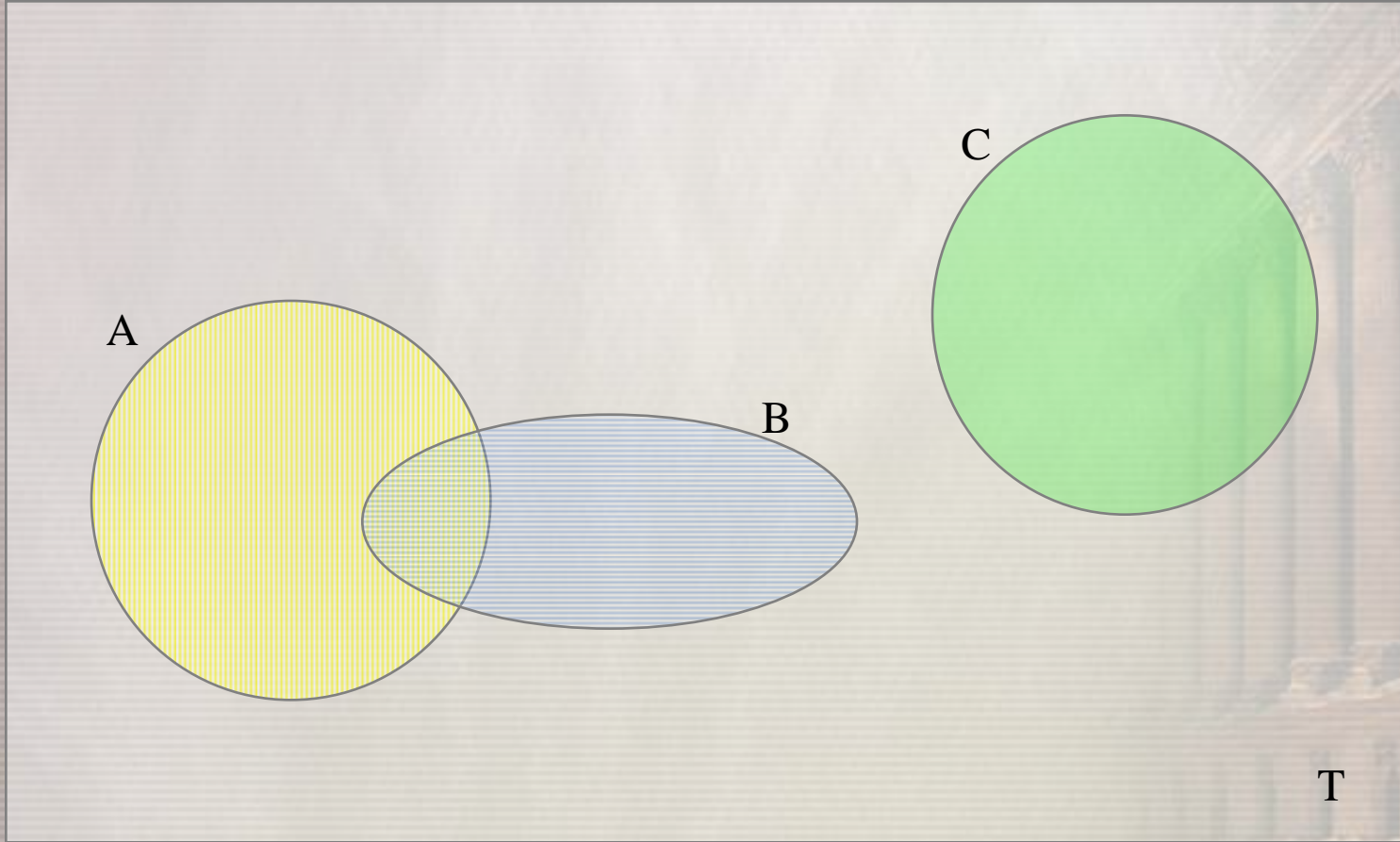
# Klasik Mantık

eşdeğer gösterimler tablosu

Kümeler Teorisi	Boolean Cebiri	Önermeler Mantığı
$P(x)$	$B$	$P(v)$
$U$	$+$	$\vee$
$\cap$	$\cdot$	$\wedge$
$-$	$-$	$\neg$
$x$	$1$	$1$
$\emptyset$	$0$	$0$
$\cup$	$\leq$	$\Rightarrow$

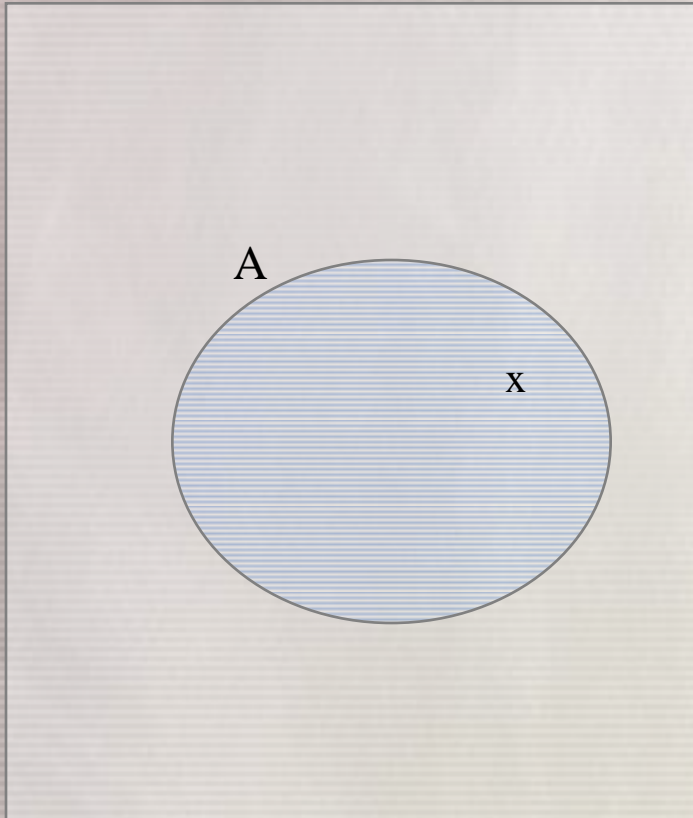
# Klasik Kümeler

venn diyagramı

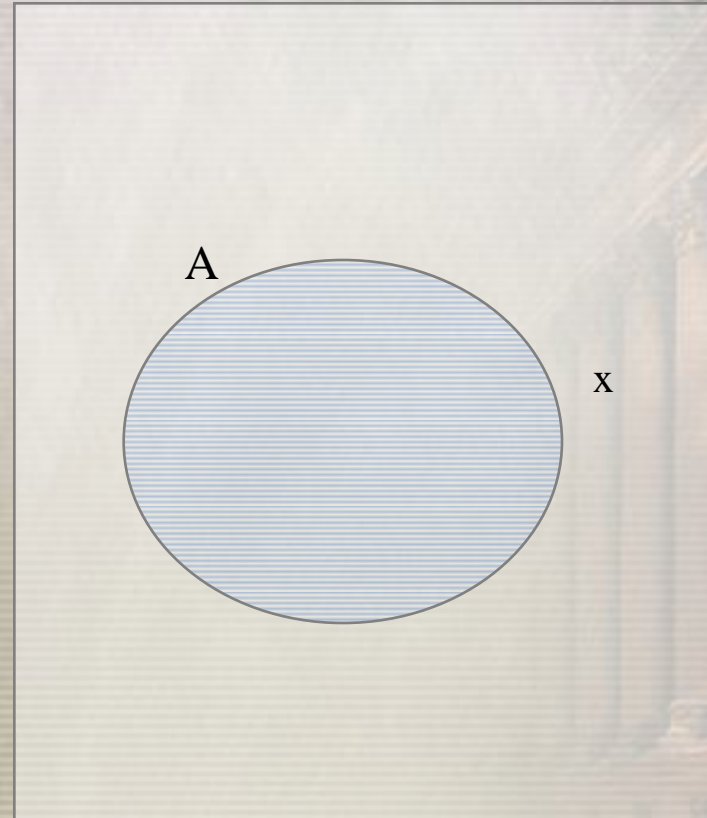


# Klasik Kümeler

eleman olma durumu



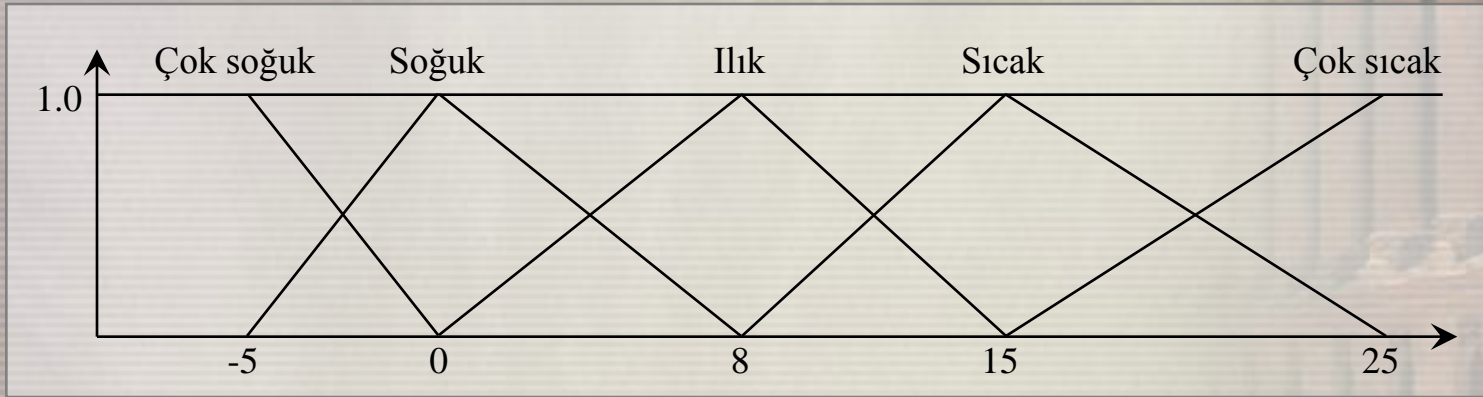
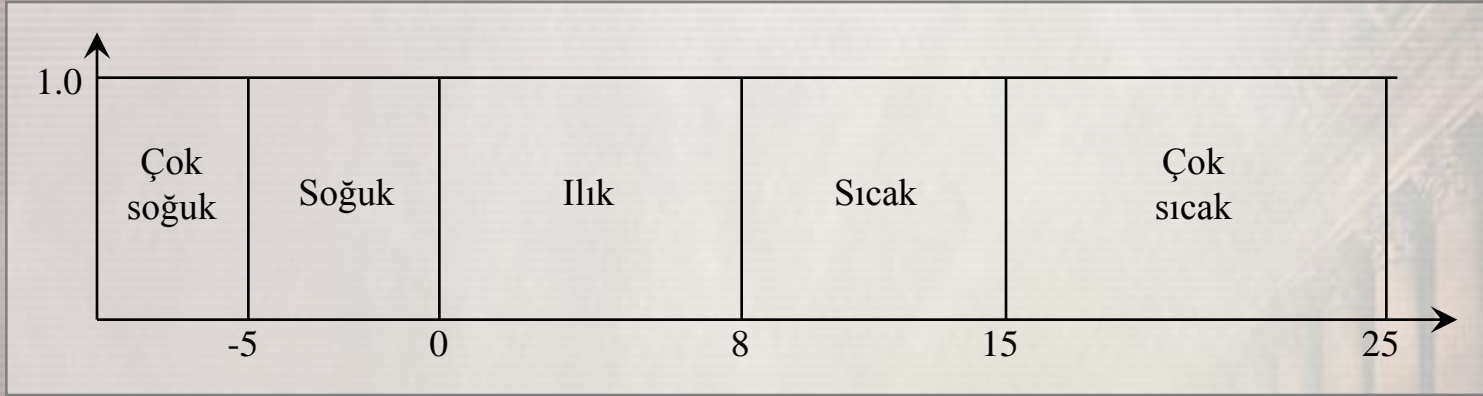
$x \in A$



$x \notin A$

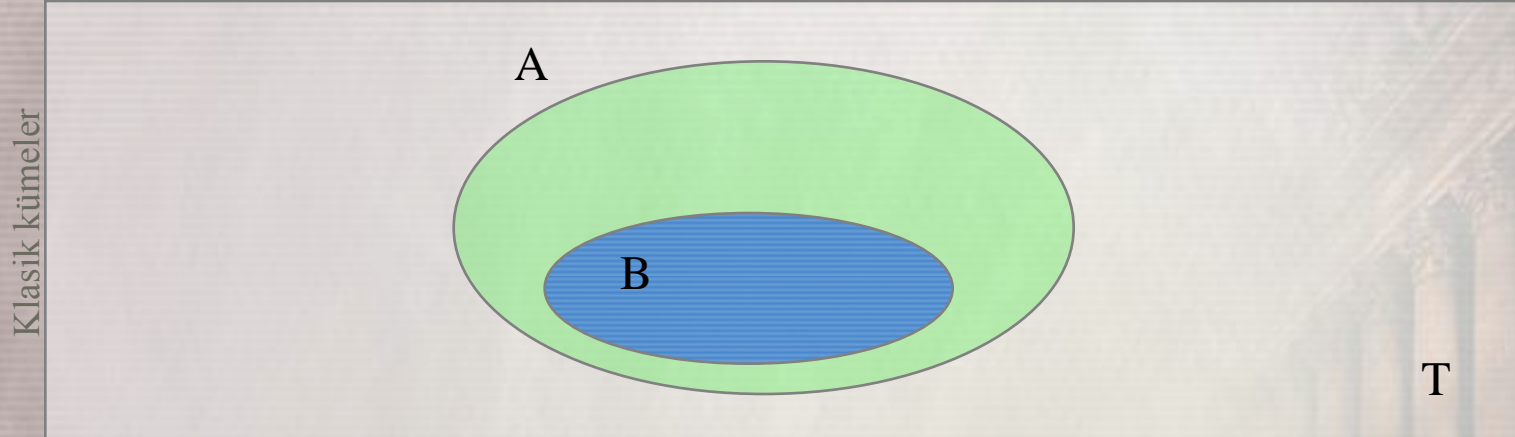
# Bulanık Kümeler

eleman olma durumu

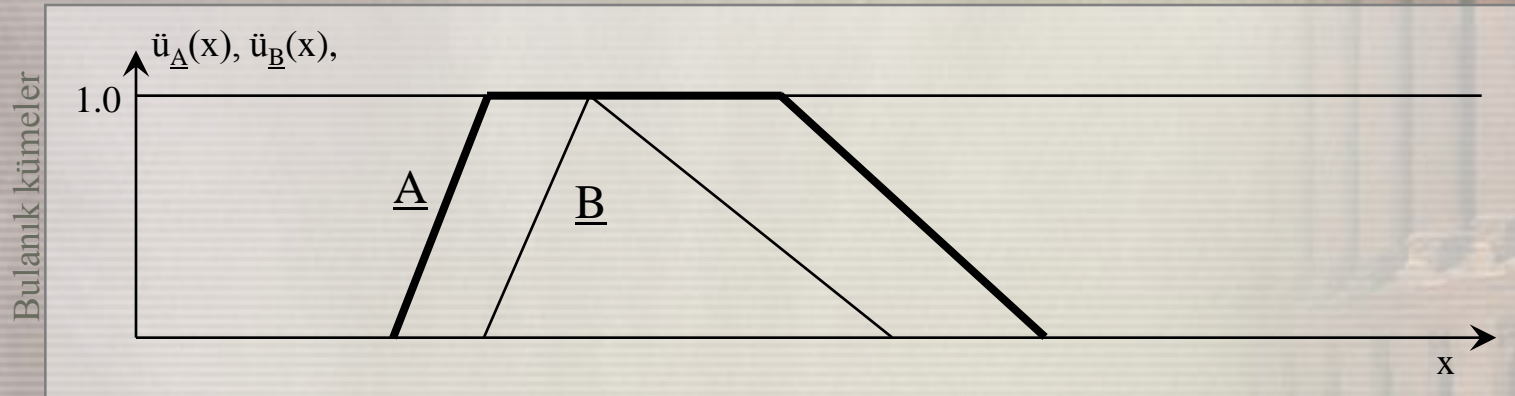


# Klasik ve Bulanık Kümeler

alt küme

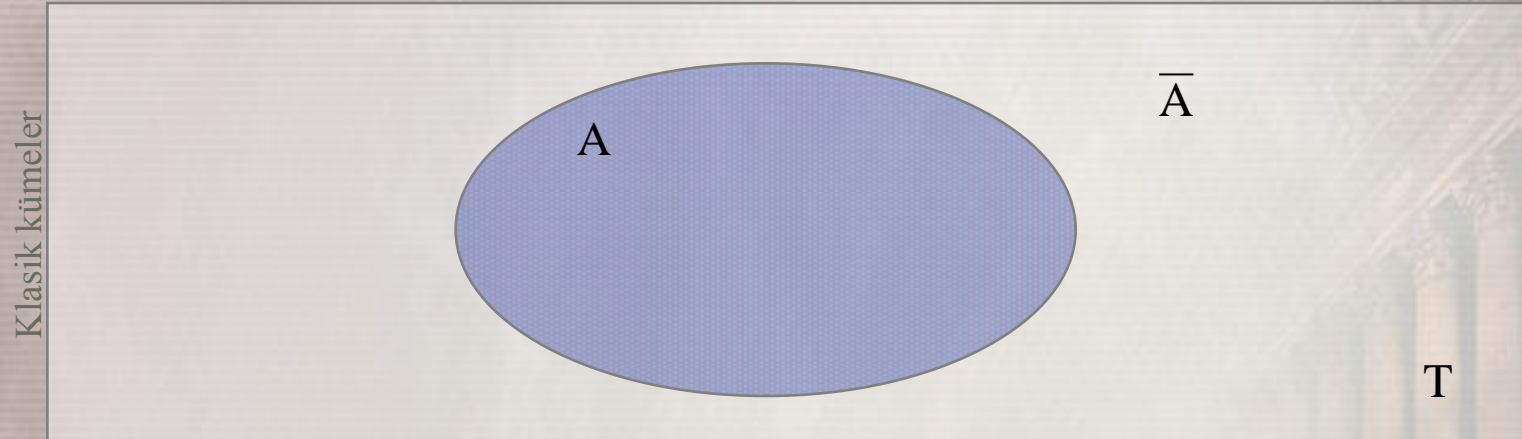


$$A \subseteq B \text{ ve } \underline{A} \subseteq \underline{B}$$

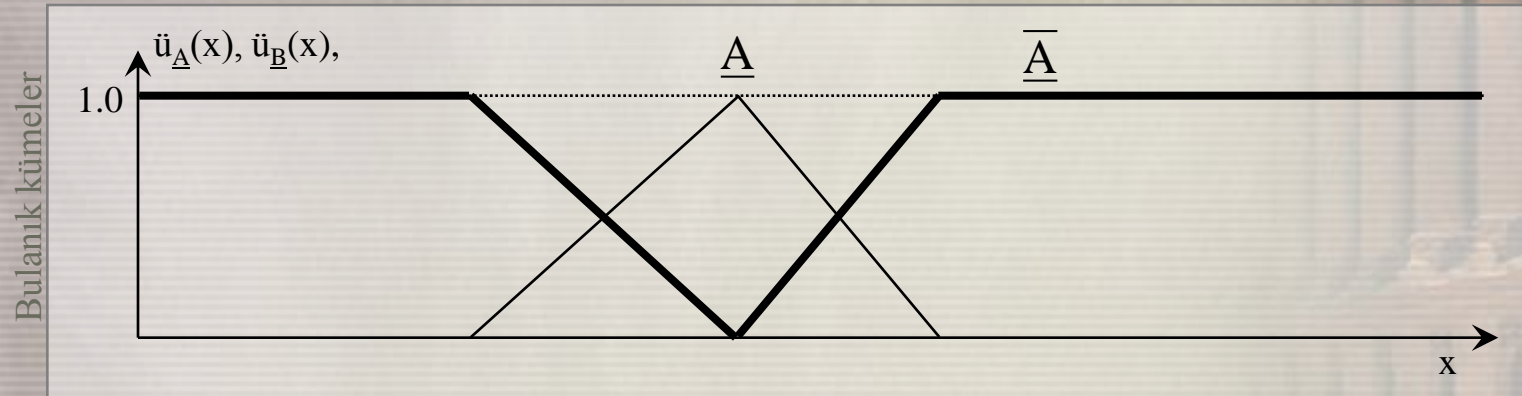


# Klasik ve Bulanık Kümeler

tamamlayıcı kümeler

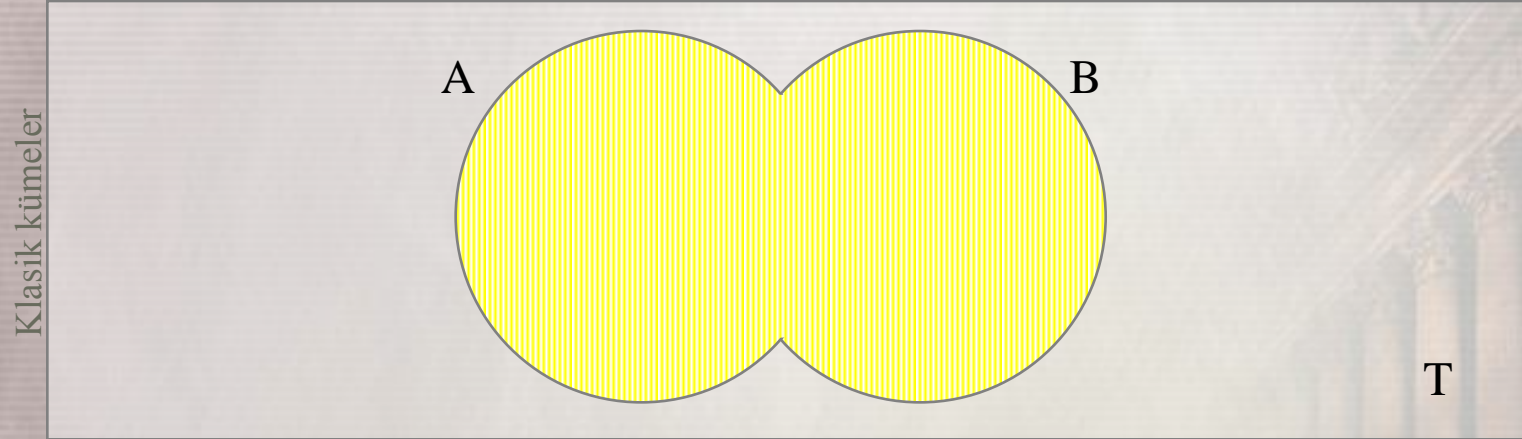


$$A \cup \bar{A} = T \text{ ve } \underline{A} \vee \bar{\underline{A}} = T$$

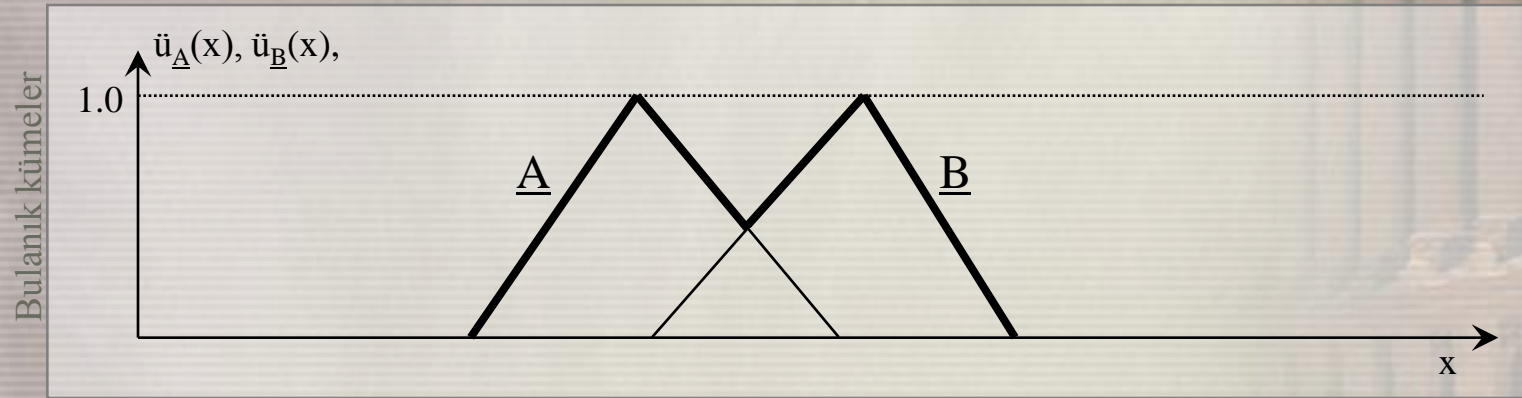


# Klasik ve Bulanık Kümeler

birleşim kümeleri



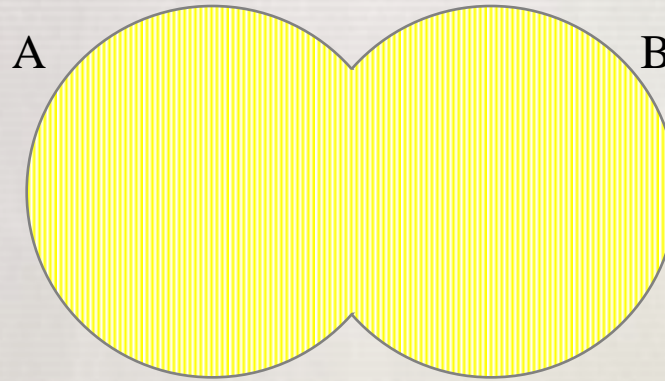
$A \cup B$  ve  $\underline{A} \vee \underline{B}$



# Klasik ve Bulanık Kümeler

klasik kümelerde birleşim

Klasik kümeler



$A \cup B$

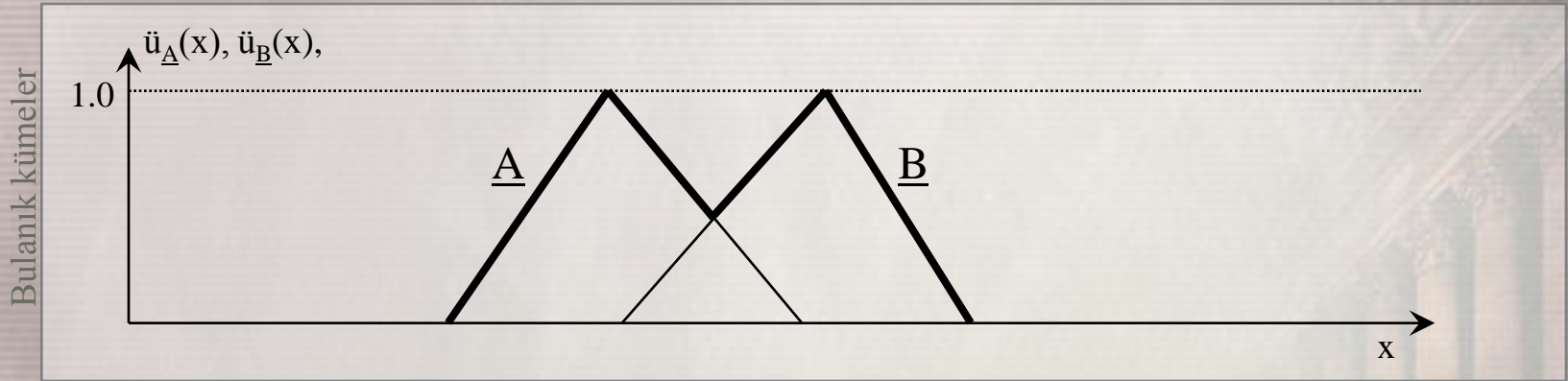
$$A = \{a, b, c, e, h, i\}$$

$$B = \{1, a, 3, z, b, x\}$$

$$A \cup B = \{a, 1, 3, b, c, e, h, i, z, x\}$$

# Klasik ve Bulanık Kümeler

bulanık kümelerde birleşim



$\underline{A} \vee \underline{B}$

$$\underline{A} = \{0.1/a + 0.3/b + 0.9/c + 1.0/e + 0.6/h + 0.2/i\}$$

$$\underline{B} = \{1.0/1 + 0.8/a + 0.6/3 + 0.4/z + 0.2/b + 0.1/x\}$$

$$\underline{A} \vee \underline{B} = \{0.8/a + 1.0/1 + 0.6/3 + 0.3/b + 0.9/c + 1.0/e + 0.6/h + 0.2/i + 0.4/z + 0.1/x\}$$

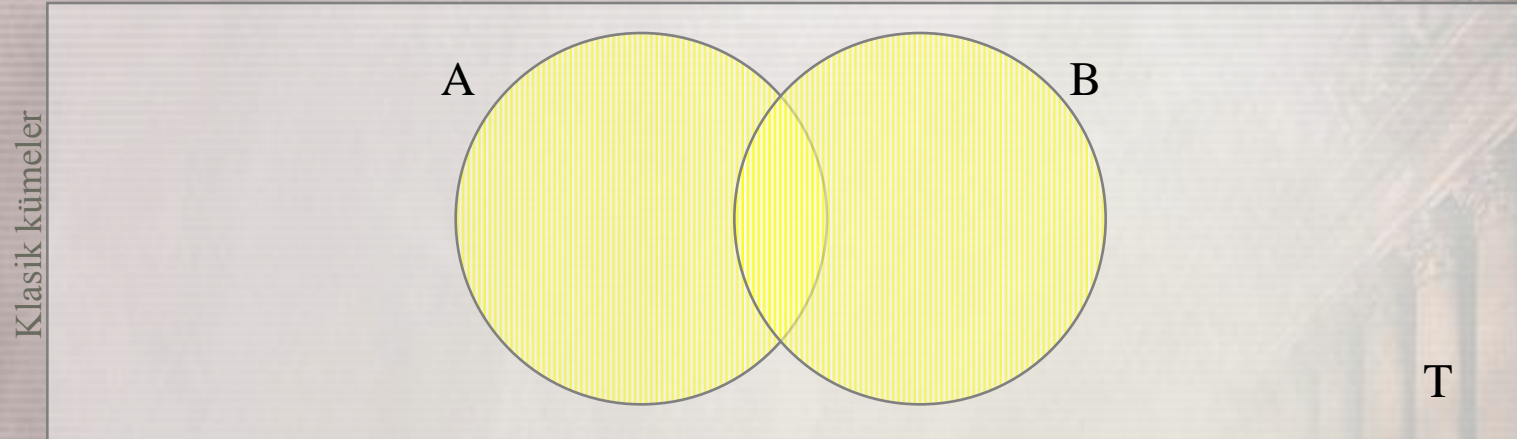
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$$\ddot{u}(a) = EB[0.1, 0.8] = 0.8$$

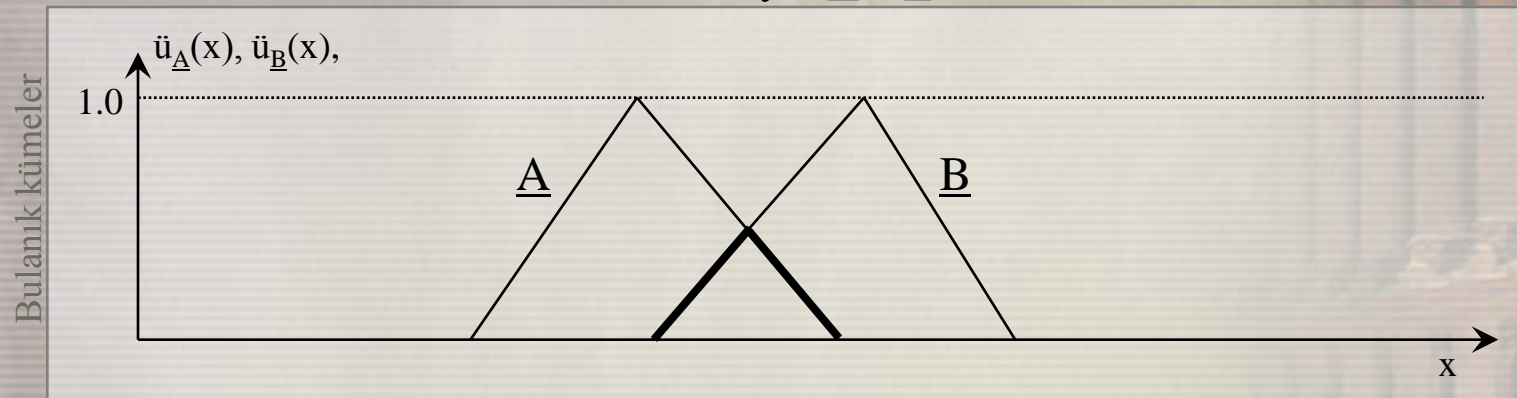
$$\ddot{u}(b) = EB[0.3, 0.2] = 0.3$$

# Klasik ve Bulanık Kümeler

kesişim kümeleri



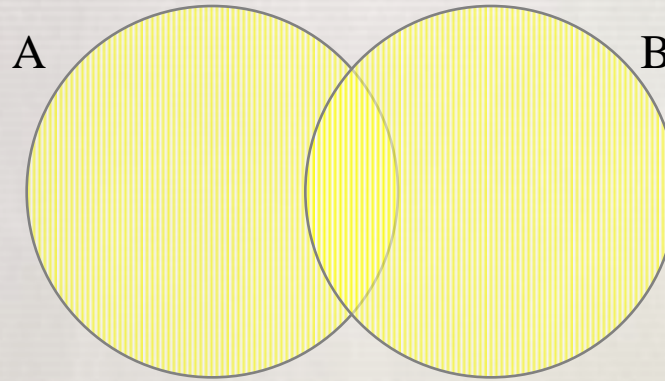
$A \cap B$  veya  $\underline{A} \wedge \underline{B}$



# Klasik ve Bulanık Kümeler

klasik kümelerde kesişim

Klasik kümeler



$A \cap B$

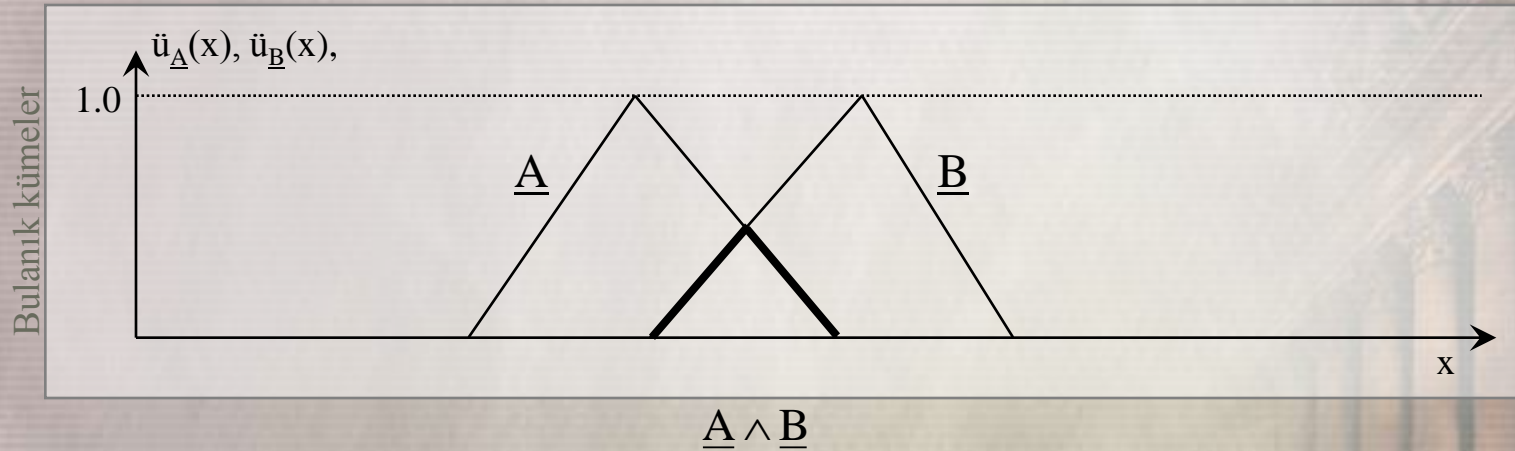
$$A = \{a, b, c, e, h, i\}$$

$$B = \{1, a, 3, z, b, x\}$$

$$A \cap B = \{a, b\}$$

# Klasik ve Bulanık Kümeler

bulanık kümelerde kesişim



$$\underline{A} = \{0.1/a + 0.3/b + 0.9/c + 1.0/e + 0.6/h + 0.2/i\}$$

$$\underline{B} = \{1.0/1 + 0.8/a + 0.6/3 + 0.4/z + 0.2/b + 0.1/x\}$$

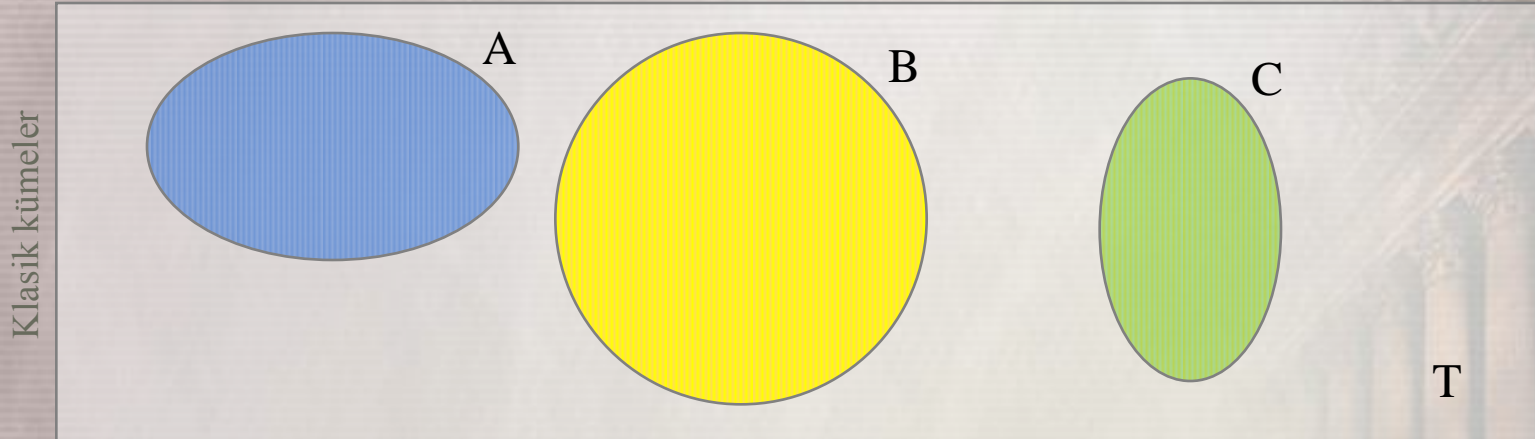
$$\underline{A} \wedge \underline{B} = \{0.1/a + 0.2/b\}$$

$$\ddot{u}(a) = \text{EK}[0.1, 0.8] = 0.1$$

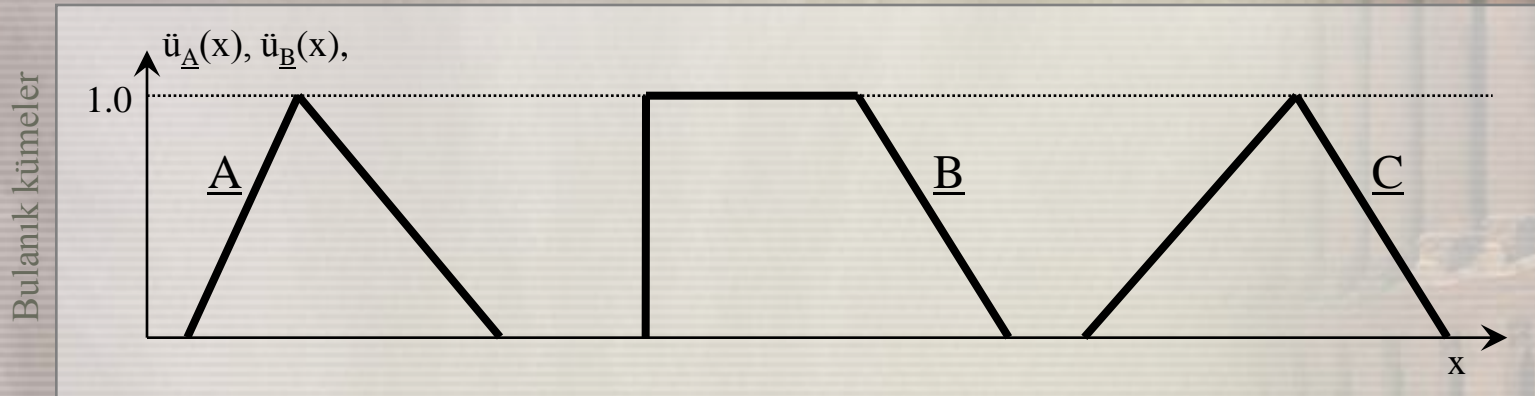
$$\ddot{u}(b) = \text{EK}[0.3, 0.2] = 0.2$$

# Klasik ve Bulanık Kümeler

ayrık kümeler

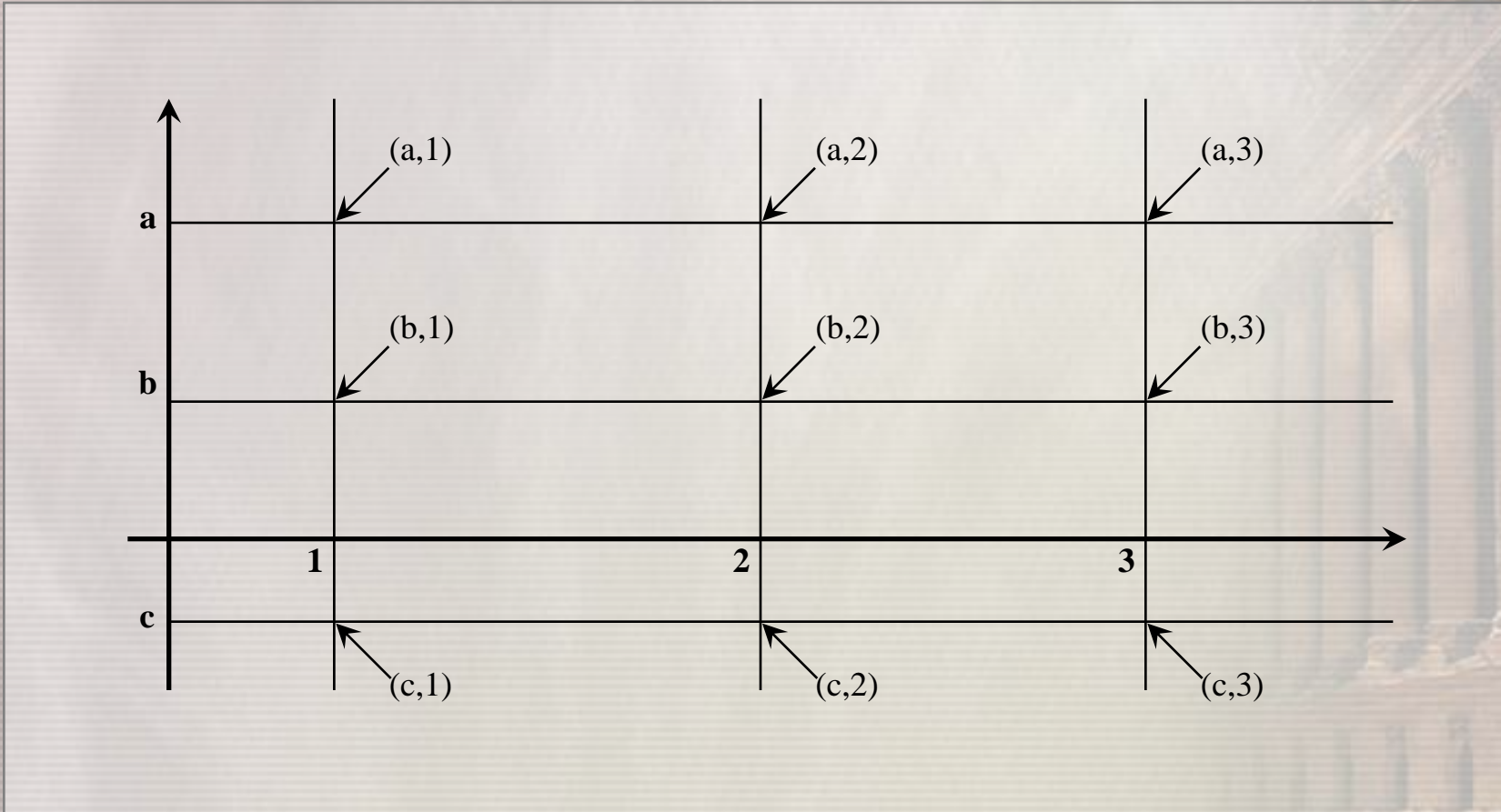


$$A \cap B = \emptyset \text{ veya } \underline{A} \wedge \underline{B} = \emptyset \quad A \cap C = \emptyset \text{ veya } \underline{A} \wedge \underline{C} = \emptyset \quad B \cap C = \emptyset \text{ veya } \underline{B} \wedge \underline{C} = \emptyset$$



# Klasik ve Bulanık Kümeler

çarpım kümesi



# Klasik ve Bulanık Kümeler

çarpım kümeleri

Klasik kümeler

$$A = \{a, b, c\}$$

$$B = \{1, 2, 3\}$$

$$C = A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$$

Bulanık kümeler

$$\underline{A} = \{0.1/a, 1.0/b, 0.3/c\}$$

$$\underline{B} = \{0.3/1+0.7/2+1.0/3\}$$

$$\underline{A} \wedge \underline{B} = \{0.1/(1,a) + 0.3/(1,b) + 0.3/(1,c) + 0.1/(2,a) + 0.7/(2,b) + 0.3/(2,c) \\ + 0.1/(3,a) + 1.0/(3,b) + 0.3/3,c)\}$$

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$$\mu_{\underline{A} \times \underline{B}}(x) = EK[\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)]$$

# Klasik ve Bulanık Kümeler

çoklu küme işlemleri

Birleşme özelliği

$$A \cup B = B \cup A = \underline{A} \vee \underline{B} = \underline{B} \vee \underline{A}$$

$$A \cap B = B \cap A = \underline{A} \wedge \underline{B} = \underline{B} \wedge \underline{A}$$

$$(A \cup B) \cup C = A \cup (B \cup C) = (\underline{A} \vee \underline{B}) \vee \underline{C} = \underline{A} \vee (\underline{B} \vee \underline{C})$$

$$(A \cap B) \cap C = A \cap (B \cap C) = (\underline{A} \wedge \underline{B}) \wedge \underline{C} = \underline{A} \wedge (\underline{B} \wedge \underline{C})$$

Dağılıma özelliği

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \underline{A} \wedge (\underline{B} \vee \underline{C}) = (\underline{A} \wedge \underline{B}) \vee (\underline{A} \wedge \underline{C})$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \underline{A} \vee (\underline{B} \wedge \underline{C}) = (\underline{A} \vee \underline{B}) \wedge (\underline{A} \vee \underline{C})$$

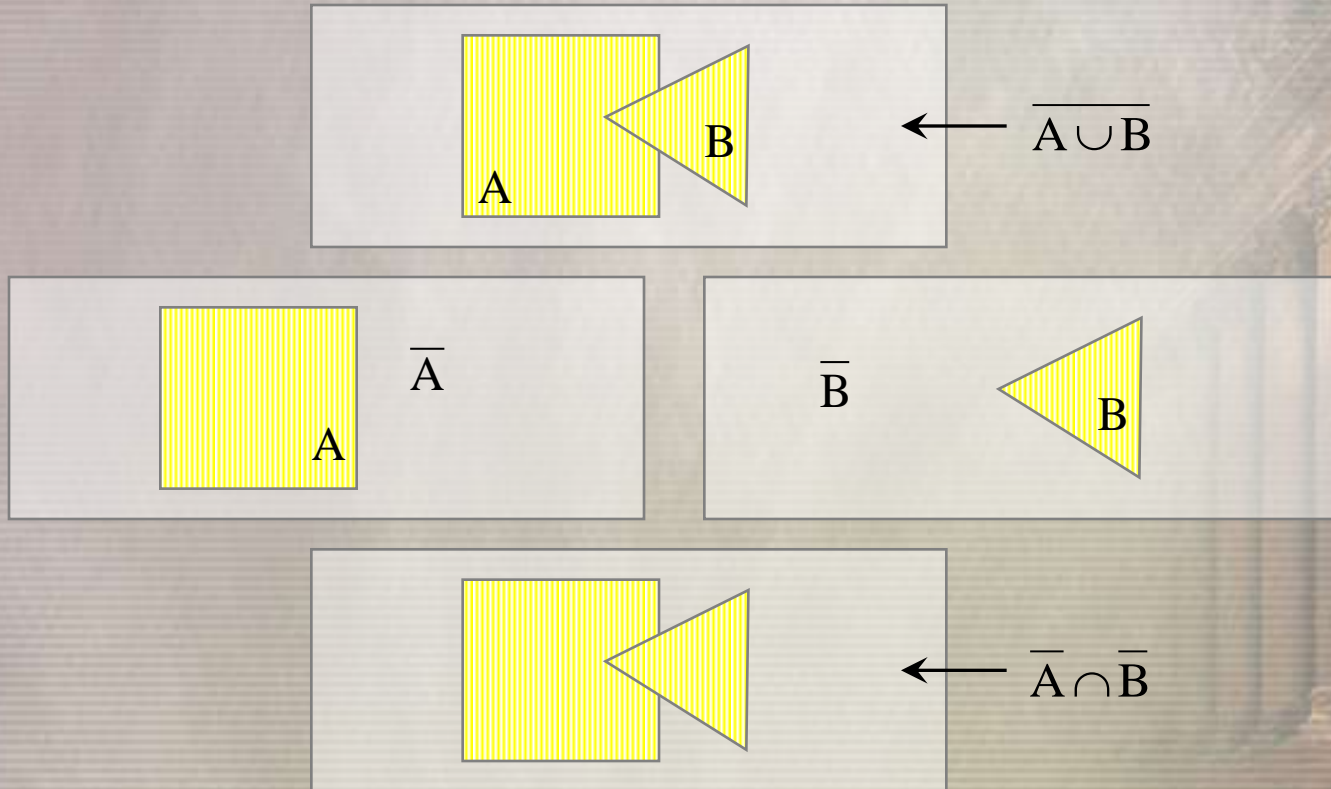
De Morgan kuralı

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

# Klasik ve Bulanık Kümeler

De Morgan kuralı



# Teşekkürler



İstanbul Üniversitesi İnşaat Mühendisliği Bölümü