

Mukavemet - özümlü Örnekler

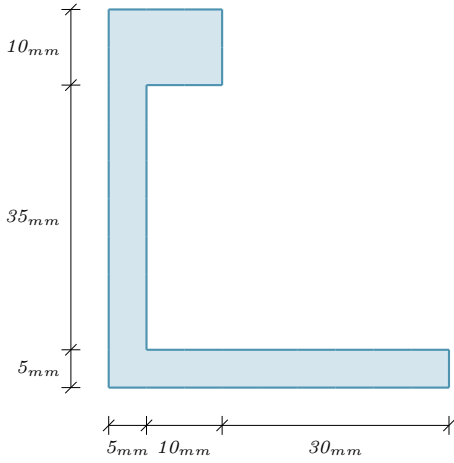
Dr. Rasim Temur

Mayıs 2015

Bölüm 2

Kesit Atalet Momenti

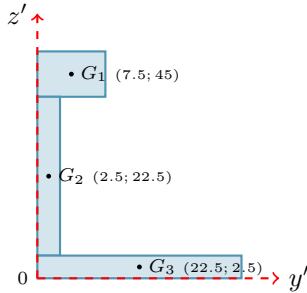
2.1 Örnek



Şekildeki kesitte;

- I_y , I_z , I_{yz} ve I_0 atalet momentlerini bulunuz;
- i_y , i_z ve i_0 atalet yarıçaplarını bulunuz;
- Asal atalet momentlerini hesaplayınız;
- Asal eksen takımının konumunu belirleyiniz ve şekil üzerinde gösteriniz.

Alan ağırlık merkezi :



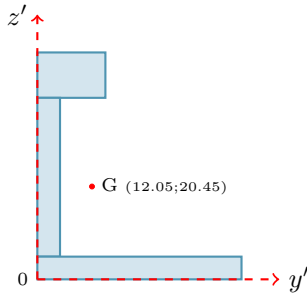
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$):

$$G_{y'} = \frac{A_1 \cdot y'_1 + A_2 \cdot y'_2 + A_3 \cdot y'_3}{A_1 + A_2 + A_3}$$

$$G_{y'} = \frac{150 \cdot 7.5 + 175 \cdot 2.5 + 225 \cdot 22.5}{150 + 175 + 225}$$

$$G_{y'} = \frac{6625}{550}$$

$$G_{y'} = 12.05 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$):

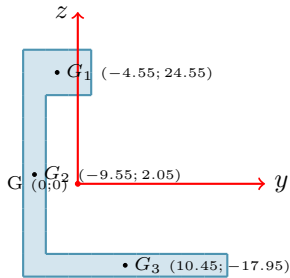
$$G_{z'} = \frac{A_1 \cdot z'_1 + A_2 \cdot z'_2 + A_3 \cdot z'_3}{A_1 + A_2 + A_3}$$

$$G_{z'} = \frac{150 \cdot 45 + 175 \cdot 22.5 + 225 \cdot 2.5}{150 + 175 + 225}$$

$$G_{z'} = \frac{11250}{550}$$

$$G_{z'} = 20.45 \text{ mm}$$

a. I_y , I_z , I_{yz} ve I_0 atalet momentleri :



Eksenlere olan uzaklıklar :

$$G_1 (-4.55; 24.55) \Rightarrow y_1 = -4.55 \text{ (mm)} \quad z_1 = 24.55 \text{ (mm)}$$

$$G_2 (-9.55; 2.05) \Rightarrow y_2 = -9.55 \text{ (mm)} \quad z_2 = 2.05 \text{ (mm)}$$

$$G_3 (10.45; -17.95) \Rightarrow y_3 = 10.45 \text{ (mm)} \quad z_3 = -17.95 \text{ (mm)}$$

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot z_1^2 = \frac{15 \cdot 10^3}{12} + 150 \cdot (24.55)^2 \Rightarrow I_y^1 = 91655.38 \text{ mm}^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot z_2^2 = \frac{5 \cdot 35^3}{12} + 175 \cdot (2.05)^2 \Rightarrow I_y^2 = 18600.02 \text{ mm}^4$$

$$I_y^3 = \frac{b_3 \cdot h_3^3}{12} + A_3 \cdot z_3^2 = \frac{45 \cdot 5^3}{12} + 225 \cdot (-17.95)^2 \Rightarrow I_y^3 = 72964.31 \text{ mm}^4$$

$$I_y = I_y^1 + I_y^2 + I_y^3$$

$$I_y = 91655.38 + 18600.02 + 72964.31 \Rightarrow I_y = 183220 \text{ mm}^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot y_1^2 = \frac{10 \cdot 15^3}{12} + 150 \cdot (-4.55)^2 \Rightarrow I_z^1 = 5917.88 \text{ mm}^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot y_2^2 = \frac{35 \cdot 5^3}{12} + 175 \cdot (-9.55)^2 \Rightarrow I_z^2 = 16325.02 \text{ mm}^4$$

$$I_z^3 = \frac{h_3 \cdot b_3^3}{12} + A_3 \cdot y_3^2 = \frac{5 \cdot 45^3}{12} + 225 \cdot (10.45)^2 \Rightarrow I_z^3 = 62539.31 \text{ mm}^4$$

$$I_z = I_z^1 + I_z^2 + I_z^3$$

$$I_z = 5917.88 + 16325.02 + 62539.31 \Rightarrow I_z = 84782 \text{ mm}^4$$

ζ arpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 150 \cdot (-4.55) \cdot 24.55 \Rightarrow I_{yz}^1 = -16721.25 \text{ mm}^4$$

$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 175 \cdot (-9.55) \cdot 2.05 \Rightarrow I_{yz}^2 = -3342.5 \text{ mm}^4$$

$$I_{yz}^3 = A_3 \cdot y_3 \cdot z_3 = 225 \cdot 10.45 \cdot (-17.95) \Rightarrow I_{yz}^3 = -42322.5 \text{ mm}^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2 + I_{yz}^3$$

$$I_{yz} = -16721 - 3342.5 - 42322.5 \Rightarrow I_{yz} = -62386 \text{ mm}^4$$

Kutupsal atalet momenti (I_0) :

$$I_0 = I_y + I_z$$

$$I_0 = 183220 + 84782 \Rightarrow I_0 = 268002 \text{ mm}^4$$

b. Atalet yarıçapları (i_y , i_z ve i_0) :

$$i_y = \sqrt{\frac{I_y}{A}} \quad i_y = \sqrt{\frac{183220}{550}} \Rightarrow i_y = 18.3 \text{ mm}$$

$$i_z = \sqrt{\frac{I_z}{A}} \quad i_z = \sqrt{\frac{84782}{550}} \Rightarrow i_z = 12.4 \text{ mm}$$

$$i_0 = \sqrt{\frac{I_0}{A}} \quad i_0 = \sqrt{\frac{268002}{550}} \Rightarrow i_0 = 22.1 \text{ mm}$$

c. Asal atalet momentleri ($I_{max,min}$) :

$$I_{max,min} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

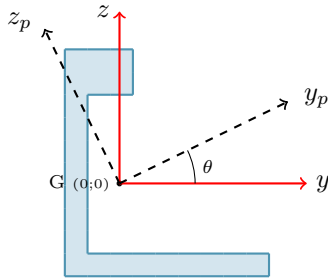
$$I_{max,min} = \frac{183220 + 84782}{2} \pm \sqrt{\left(\frac{183220 - 84782}{2}\right)^2 + (-62386)^2}$$

$$I_{max,min} = \frac{268002}{2} \pm \sqrt{\left(\frac{98438}{2}\right)^2 + 3892012996}$$

$$I_{max,min} = 134001 \pm 79463.97$$

$$I_{max} = 213465 \text{ mm}^4$$

$$I_{min} = 54537 \text{ mm}^4$$

d. Asal eksenlerin konumu (θ) :

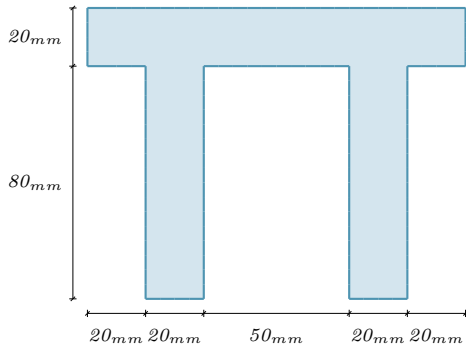
$$\tan 2\theta = -\frac{2 \cdot I_{yz}}{I_y - I_z}$$

$$\tan 2\theta = -\frac{2 \cdot (-62386)}{183220 - 84782}$$

$$\tan 2\theta = 1.27$$

$$2\theta = 51.8^\circ \Rightarrow \theta = 25.9^\circ$$

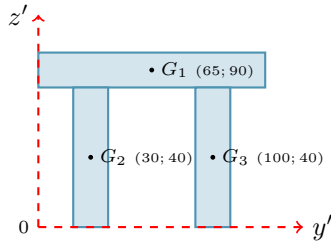
2.2 Örnek



Şekildeki kesitte;

- I_y , I_z , I_{yz} ve I_0 atalet momentlerini bulunuz;
- i_y , i_z ve i_0 atalet yarıçaplarını bulunuz;
- Asal atalet momentlerini hesaplayınız;
- Asal eksen takımının konumunu belirleyiniz ve şekil üzerinde gösteriniz.

Alan ağırlık merkezi:



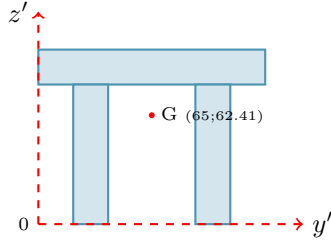
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$):

$$G_{y'} = \frac{A_1 \cdot y'_1 + A_2 \cdot y'_2 + A_3 \cdot y'_3}{A_1 + A_2 + A_3}$$

$$G_{y'} = \frac{2600 \cdot 65 + 1600 \cdot 30 + 1600 \cdot 100}{2600 + 1600 + 1600}$$

$$G_{y'} = \frac{377000}{5800}$$

$$G_{y'} = 65 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$):

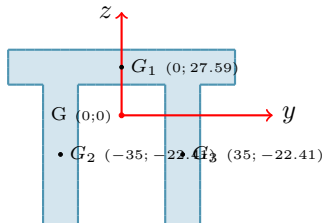
$$G_{z'} = \frac{A_1 \cdot z'_1 + A_2 \cdot z'_2 + A_3 \cdot z'_3}{A_1 + A_2 + A_3}$$

$$G_{z'} = \frac{2600 \cdot 90 + 1600 \cdot 40 + 1600 \cdot 40}{2600 + 1600 + 1600}$$

$$G_{z'} = \frac{362000}{5800}$$

$$G_{z'} = 62.41 \text{ mm}$$

a. I_y , I_z , I_{yz} ve I_0 atalet momentleri:



Eksenlere olan uzaklıklar:

$$G_1 (0; 27.59) \Rightarrow y_1 = 0 \text{ (mm)} \quad z_1 = 27.59 \text{ (mm)}$$

$$G_2 (-35; -22.41) \Rightarrow y_2 = -35 \text{ (mm)} \quad z_2 = -22.41 \text{ (mm)}$$

$$G_3 (35; -22.41) \Rightarrow y_3 = 35 \text{ (mm)} \quad z_3 = -22.41 \text{ (mm)}$$

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot z_1^2 = \frac{130 \cdot 20^3}{12} + 2600 \cdot (27.59)^2 \Rightarrow I_y^1 = 2065807.75 \text{ mm}^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot z_2^2 = \frac{20 \cdot 80^3}{12} + 1600 \cdot (-22.41)^2 \Rightarrow I_y^2 = 1656866.25 \text{ mm}^4$$

$$I_y^3 = \frac{b_3 \cdot h_3^3}{12} + A_3 \cdot z_3^2 = \frac{20 \cdot 80^3}{12} + 1600 \cdot (-22.41)^2 \Rightarrow I_y^3 = 1656866.25 \text{ mm}^4$$

$$I_y = I_y^1 + I_y^2 + I_y^3$$

$$I_y = 2065807.75 + 1656866.25 + 1656866.25 \Rightarrow I_y = 5379540 \text{ mm}^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot y_1^2 = \frac{20 \cdot 130^3}{12} + 2600 \cdot (0)^2 \Rightarrow I_z^1 = 3661666.75 \text{ mm}^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot y_2^2 = \frac{80 \cdot 20^3}{12} + 1600 \cdot (-35)^2 \Rightarrow I_z^2 = 2013333.38 \text{ mm}^4$$

$$I_z^3 = \frac{h_3 \cdot b_3^3}{12} + A_3 \cdot y_3^2 = \frac{80 \cdot 20^3}{12} + 1600 \cdot (35)^2 \Rightarrow I_z^3 = 2013333.38 \text{ mm}^4$$

$$I_z = I_z^1 + I_z^2 + I_z^3$$

$$I_z = 3661666.75 + 2013333.38 + 2013333.38 \Rightarrow I_z = 7688334 \text{ mm}^4$$

Çarpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 2600 \cdot 0 \cdot 27.59 \Rightarrow I_{yz}^1 = 0 \text{ mm}^4$$

$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 1600 \cdot (-35) \cdot (-22.41) \Rightarrow I_{yz}^2 = 1254400 \text{ mm}^4$$

$$I_{yz}^3 = A_3 \cdot y_3 \cdot z_3 = 1600 \cdot 35 \cdot (-22.41) \Rightarrow I_{yz}^3 = -1254400 \text{ mm}^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2 + I_{yz}^3$$

$$I_{yz} = 0 + 1254400 - 1254400 \Rightarrow I_{yz} = 0 \text{ mm}^4$$

Kutupsal atalet momenti (I_0) :

$$I_0 = I_y + I_z$$

$$I_0 = 5379540 + 7688334 \Rightarrow I_0 = 13067874 \text{ mm}^4$$

b. Atalet yarıçapları (i_y , i_z ve i_0) :

$$i_y = \sqrt{\frac{I_y}{A}} \quad i_y = \sqrt{\frac{5379540}{5800}} \Rightarrow i_y = 30.5 \text{ mm}$$

$$i_z = \sqrt{\frac{I_z}{A}} \quad i_z = \sqrt{\frac{7688334}{5800}} \Rightarrow i_z = 36.4 \text{ mm}$$

$$i_0 = \sqrt{\frac{I_0}{A}} \quad i_0 = \sqrt{\frac{13067874}{5800}} \Rightarrow i_0 = 47.5 \text{ mm}$$

c. Asal atalet momentleri ($I_{max,min}$) :

$$I_{max,min} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

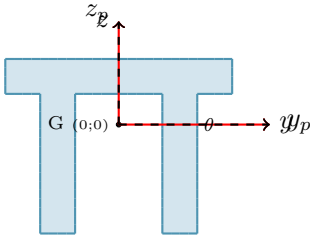
$$I_{max,min} = \frac{5379540 + 7688334}{2} \pm \sqrt{\left(\frac{5379540 - 7688334}{2}\right)^2 + 0^2}$$

$$I_{max,min} = \frac{13067874}{2} \pm \sqrt{\left(\frac{-2308794}{2}\right)^2 + 0}$$

$$I_{max,min} = 6533937 \pm 1154397$$

$$I_{max} = 7688334 \text{ mm}^4$$

$$I_{min} = 5379540 \text{ mm}^4$$

d. Asal eksenlerin konumu (θ) :

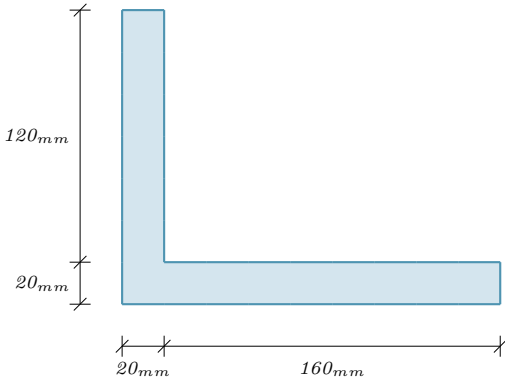
$$\tan 2\theta = -\frac{2 \cdot I_{yz}}{I_y - I_z}$$

$$\tan 2\theta = -\frac{2 \cdot 0}{5379540 - 7688334}$$

$$\tan 2\theta = 0$$

$$2\theta = 0^\circ \Rightarrow \theta = 0^\circ$$

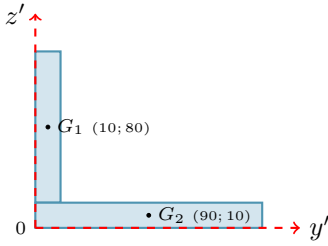
2.3 Örnek



Şekildeki kesitte;

- I_y , I_z , I_{yz} ve I_0 atalet momentlerini bulunuz;
- i_y , i_z ve i_0 atalet yarıçaplarını bulunuz;
- Asal atalet momentlerini hesaplayınız;
- Asal eksen takımının konumunu belirleyiniz ve şekil üzerinde gösteriniz.

Alan ağırlık merkezi :



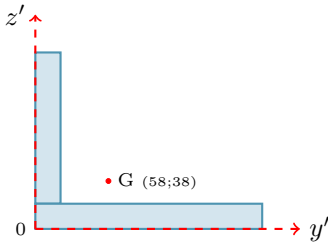
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$) :

$$G_{y'} = \frac{A_1 \cdot y'_1 + A_2 \cdot y'_2}{A_1 + A_2}$$

$$G_{y'} = \frac{2400 \cdot 10 + 3600 \cdot 90}{2400 + 3600}$$

$$G_{y'} = \frac{348000}{6000}$$

$$G_{y'} = 58 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$) :

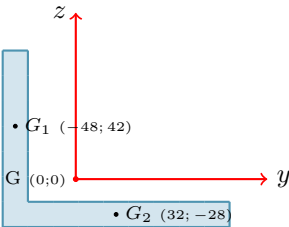
$$G_{z'} = \frac{A_1 \cdot z'_1 + A_2 \cdot z'_2}{A_1 + A_2}$$

$$G_{z'} = \frac{2400 \cdot 80 + 3600 \cdot 10}{2400 + 3600}$$

$$G_{z'} = \frac{228000}{6000}$$

$$G_{z'} = 38 \text{ mm}$$

a. I_y , I_z , I_{yz} ve I_0 atalet momentleri :



Eksenlere olan uzaklıklar :

$$G_1 (-48; 42) \Rightarrow y_1 = -48 \text{ (mm)} \quad z_1 = 42 \text{ (mm)}$$

$$G_2 (32; -28) \Rightarrow y_2 = 32 \text{ (mm)} \quad z_2 = -28 \text{ (mm)}$$

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot z_1^2 = \frac{20 \cdot 120^3}{12} + 2400 \cdot (42)^2 \Rightarrow I_y^1 = 7113600 \text{ mm}^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot z_2^2 = \frac{180 \cdot 20^3}{12} + 3600 \cdot (-28)^2 \Rightarrow I_y^2 = 2942400 \text{ mm}^4$$

$$I_y = I_y^1 + I_y^2$$

$$I_y = 7113600 + 2942400 \Rightarrow I_y = 10056000 \text{ mm}^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot y_1^2 = \frac{120 \cdot 20^3}{12} + 2400 \cdot (-48)^2 \Rightarrow I_z^1 = 5609600 \text{ mm}^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot y_2^2 = \frac{20 \cdot 180^3}{12} + 3600 \cdot (32)^2 \Rightarrow I_z^2 = 13406400 \text{ mm}^4$$

$$I_z = I_z^1 + I_z^2$$

$$I_z = 5609600 + 13406400 \Rightarrow I_z = 19016000 \text{ mm}^4$$

Çarpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 2400 \cdot (-48) \cdot 42 \Rightarrow I_{yz}^1 = -4838400 \text{ mm}^4$$

$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 3600 \cdot 32 \cdot (-28) \Rightarrow I_{yz}^2 = -3225600 \text{ mm}^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2$$

$$I_{yz} = -4838400 - 3225600 \Rightarrow I_{yz} = -8064000 \text{ mm}^4$$

Kutupsal atalet momenti (I_0) :

$$I_0 = I_y + I_z$$

$$I_0 = 10056000 + 19016000 \Rightarrow I_0 = 29072000 \text{ mm}^4$$

b. Atalet yarıçapları (i_y , i_z ve i_0) :

$$i_y = \sqrt{\frac{I_y}{A}} \quad i_y = \sqrt{\frac{10056000}{6000}} \Rightarrow i_y = 40.9 \text{ mm}$$

$$i_z = \sqrt{\frac{I_z}{A}} \quad i_z = \sqrt{\frac{19016000}{6000}} \Rightarrow i_z = 56.3 \text{ mm}$$

$$i_0 = \sqrt{\frac{I_0}{A}} \quad i_0 = \sqrt{\frac{29072000}{6000}} \Rightarrow i_0 = 69.6 \text{ mm}$$

c. Asal atalet momentleri ($I_{max,min}$):

$$I_{max,min} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

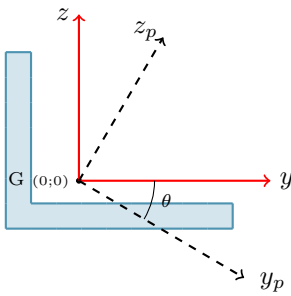
$$I_{max,min} = \frac{10056000 + 19016000}{2} \pm \sqrt{\left(\frac{10056000 - 19016000}{2}\right)^2 + (-8064000)^2}$$

$$I_{max,min} = \frac{29072000}{2} \pm \sqrt{\left(\frac{-8960000}{2}\right)^2 + 65028096000000}$$

$$I_{max,min} = 14536000 \pm 9224884.61$$

$$I_{max} = 23760885 \text{ mm}^4$$

$$I_{min} = 5311115 \text{ mm}^4$$

d. Asal eksenlerin konumu (θ):

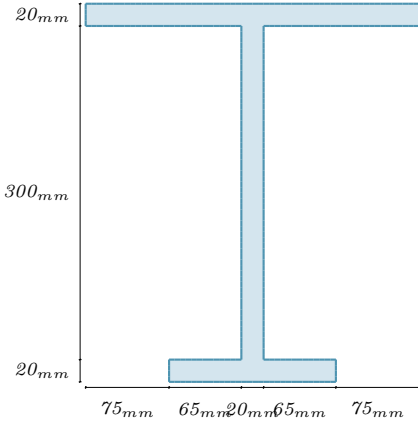
$$\tan 2\theta = -\frac{2 \cdot I_{yz}}{I_y - I_z}$$

$$\tan 2\theta = -\frac{2 \cdot (-8064000)}{10056000 - 19016000}$$

$$\tan 2\theta = -1.8$$

$$2\theta = -60.9^\circ \Rightarrow \theta = -30.5^\circ$$

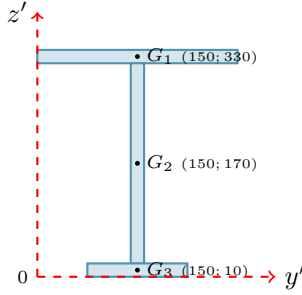
2.4 Örnek



Şekildeki kesitte;

- I_y , I_z , I_{yz} ve I_0 atalet momentlerini bulunuz;
- i_y , i_z ve i_0 atalet yarıçaplarını bulunuz;
- Asal atalet momentlerini hesaplayınız;
- Asal eksen takımının konumunu belirleyiniz ve şekil üzerinde gösteriniz.

Alan ağırlık merkezi :



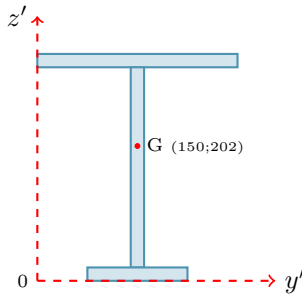
Ağırlık merkezinin z' eksenine uzaklığı ($G_{y'}$) :

$$G_{y'} = \frac{A_1 \cdot y'_1 + A_2 \cdot y'_2 + A_3 \cdot y'_3}{A_1 + A_2 + A_3}$$

$$G_{y'} = \frac{6000 \cdot 150 + 6000 \cdot 150 + 3000 \cdot 150}{6000 + 6000 + 3000}$$

$$G_{y'} = \frac{2250000}{15000}$$

$$G_{y'} = 150 \text{ mm}$$



Ağırlık merkezinin y' eksenine uzaklığı ($G_{z'}$) :

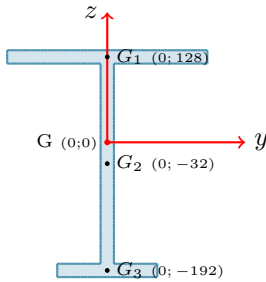
$$G_{z'} = \frac{A_1 \cdot z'_1 + A_2 \cdot z'_2 + A_3 \cdot z'_3}{A_1 + A_2 + A_3}$$

$$G_{z'} = \frac{6000 \cdot 330 + 6000 \cdot 170 + 3000 \cdot 10}{6000 + 6000 + 3000}$$

$$G_{z'} = \frac{3030000}{15000}$$

$$G_{z'} = 202 \text{ mm}$$

a. I_y , I_z , I_{yz} ve I_0 atalet momentleri :



Eksenlere olan uzaklıklar :

$$G_1 (0; 128) \Rightarrow y_1 = 0 \text{ (mm)} \quad z_1 = 128 \text{ (mm)}$$

$$G_2 (0; -32) \Rightarrow y_2 = 0 \text{ (mm)} \quad z_2 = -32 \text{ (mm)}$$

$$G_3 (0; -192) \Rightarrow y_3 = 0 \text{ (mm)} \quad z_3 = -192 \text{ (mm)}$$

I_y atalet momenti :

$$I_y = \sum \left(\frac{b \cdot h^3}{12} + A \cdot z^2 \right)$$

$$I_y^1 = \frac{b_1 \cdot h_1^3}{12} + A_1 \cdot z_1^2 = \frac{300 \cdot 20^3}{12} + 6000 \cdot (128)^2 \Rightarrow I_y^1 = 98504000 \text{ mm}^4$$

$$I_y^2 = \frac{b_2 \cdot h_2^3}{12} + A_2 \cdot z_2^2 = \frac{20 \cdot 300^3}{12} + 6000 \cdot (-32)^2 \Rightarrow I_y^2 = 51144000 \text{ mm}^4$$

$$I_y^3 = \frac{b_3 \cdot h_3^3}{12} + A_3 \cdot z_3^2 = \frac{150 \cdot 20^3}{12} + 3000 \cdot (-192)^2 \Rightarrow I_y^3 = 110692000 \text{ mm}^4$$

$$I_y = I_y^1 + I_y^2 + I_y^3$$

$$I_y = 98504000 + 51144000 + 110692000 \Rightarrow I_y = 260340000 \text{ mm}^4$$

I_z atalet momenti :

$$I_z = \sum \left(\frac{h \cdot b^3}{12} + A \cdot y^2 \right)$$

$$I_z^1 = \frac{h_1 \cdot b_1^3}{12} + A_1 \cdot y_1^2 = \frac{20 \cdot 300^3}{12} + 6000 \cdot (0)^2 \Rightarrow I_z^1 = 45000000 \text{ mm}^4$$

$$I_z^2 = \frac{h_2 \cdot b_2^3}{12} + A_2 \cdot y_2^2 = \frac{300 \cdot 20^3}{12} + 6000 \cdot (0)^2 \Rightarrow I_z^2 = 200000 \text{ mm}^4$$

$$I_z^3 = \frac{h_3 \cdot b_3^3}{12} + A_3 \cdot y_3^2 = \frac{20 \cdot 150^3}{12} + 3000 \cdot (0)^2 \Rightarrow I_z^3 = 5625000 \text{ mm}^4$$

$$I_z = I_z^1 + I_z^2 + I_z^3$$

$$I_z = 45000000 + 200000 + 5625000 \Rightarrow I_z = 50825000 \text{ mm}^4$$

Çarpım atalet momenti (I_{yz}) :

$$I_{yz} = \sum (A \cdot y \cdot z)$$

$$I_{yz}^1 = A_1 \cdot y_1 \cdot z_1 = 6000 \cdot 0 \cdot 128 \Rightarrow I_{yz}^1 = 0 \text{ mm}^4$$

$$I_{yz}^2 = A_2 \cdot y_2 \cdot z_2 = 6000 \cdot 0 \cdot (-32) \Rightarrow I_{yz}^2 = 0 \text{ mm}^4$$

$$I_{yz}^3 = A_3 \cdot y_3 \cdot z_3 = 3000 \cdot 0 \cdot (-192) \Rightarrow I_{yz}^3 = 0 \text{ mm}^4$$

$$I_{yz} = I_{yz}^1 + I_{yz}^2 + I_{yz}^3$$

$$I_{yz} = 0 + 0 + 0 \Rightarrow I_{yz} = 0 \text{ mm}^4$$

Kutupsal atalet momenti (I_0) :

$$I_0 = I_y + I_z$$

$$I_0 = 260340000 + 50825000 \Rightarrow I_0 = 311164992 \text{ mm}^4$$

b. Atalet yarıçapları (i_y , i_z ve i_0) :

$$i_y = \sqrt{\frac{I_y}{A}} \quad i_y = \sqrt{\frac{260340000}{15000}} \Rightarrow i_y = 131.7 \text{ mm}$$

$$i_z = \sqrt{\frac{I_z}{A}} \quad i_z = \sqrt{\frac{50825000}{15000}} \Rightarrow i_z = 58.2 \text{ mm}$$

$$i_0 = \sqrt{\frac{I_0}{A}} \quad i_0 = \sqrt{\frac{311164992}{15000}} \Rightarrow i_0 = 144 \text{ mm}$$

c. Asal atalet momentleri ($I_{max,min}$) :

$$I_{max,min} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

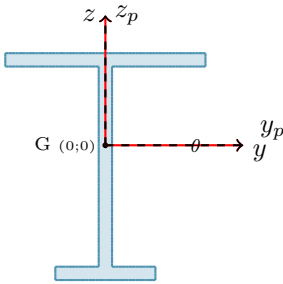
$$I_{max,min} = \frac{260340000 + 50825000}{2} \pm \sqrt{\left(\frac{260340000 - 50825000}{2}\right)^2 + 0^2}$$

$$I_{max,min} = \frac{311164992}{2} \pm \sqrt{\left(\frac{209515008}{2}\right)^2 + 0}$$

$$I_{max,min} = 155582496 \pm 104757504$$

$$I_{max} = 260340000 \text{ mm}^4$$

$$I_{min} = 50824992 \text{ mm}^4$$

d. Asal eksenlerin konumu (θ) :

$$\tan 2\theta = -\frac{2 \cdot I_{yz}}{I_y - I_z}$$

$$\tan 2\theta = -\frac{2 \cdot 0}{260340000 - 50825000}$$

$$\tan 2\theta = 0$$

$$2\theta = 0^\circ \Rightarrow \theta = 0^\circ$$